Préservation de la cohérence des transformations topologiques et géométriques

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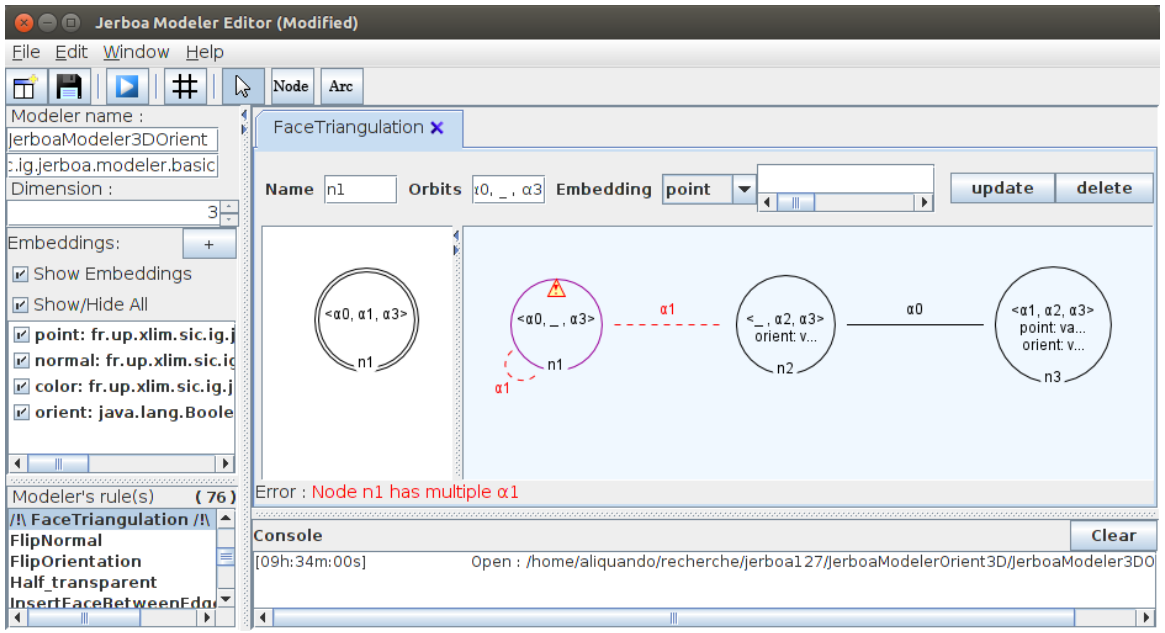
26 Novembre 2015

Introduction

Modélisation à base topologique

- Objet : graphe topologique
- Règle : transformation de graphe
- Application : transformation des objets



Sommaire

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2. Contexte
   - Objets (Cartes Généralisées)
   - Transformation topologique
   - Transformation des plongements (Géométrie)
3. Contribution
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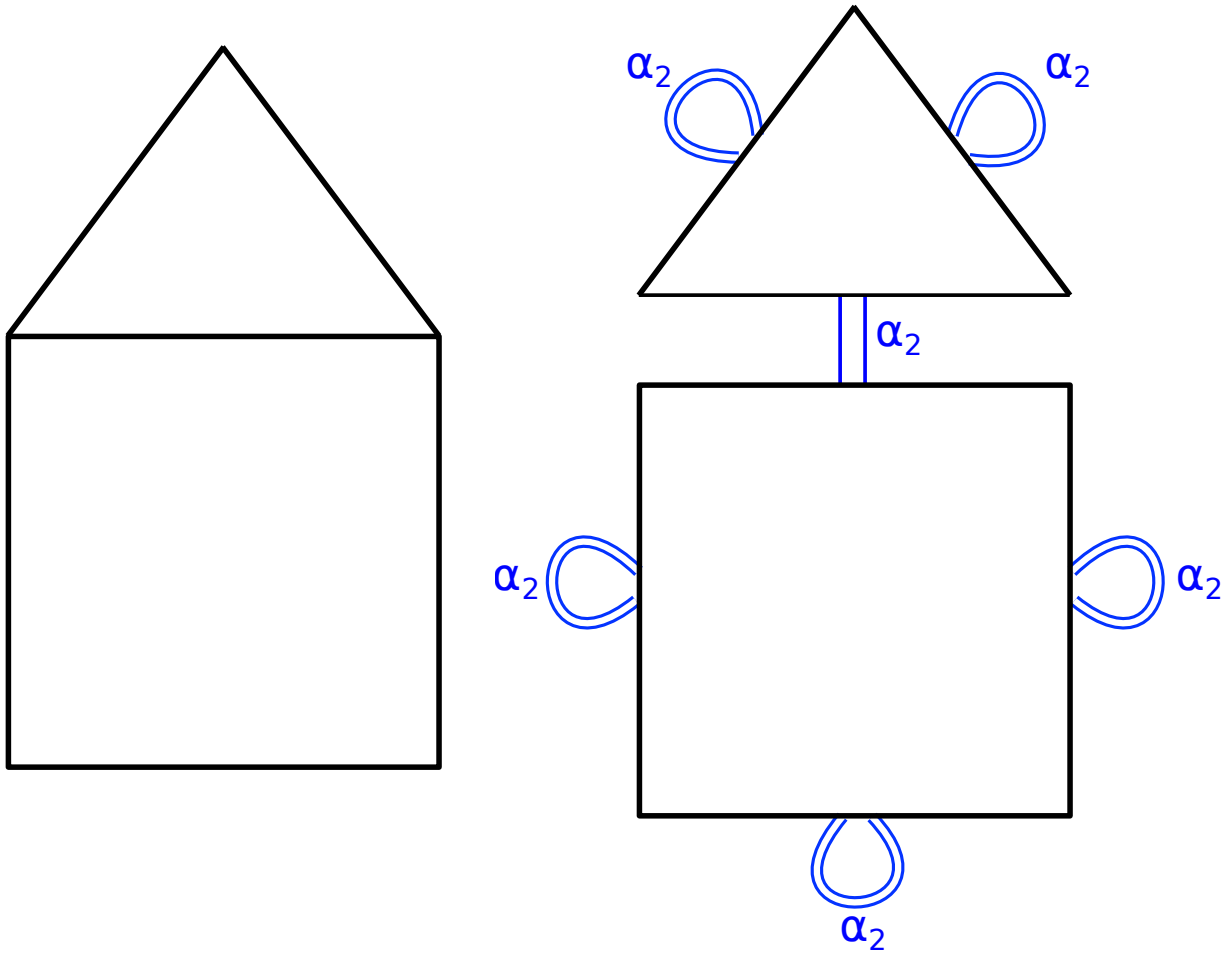
   - Condition de préservation de cohérence des plongements
4. Conclusion

Objets

Exemple de 2G-carte



Objets

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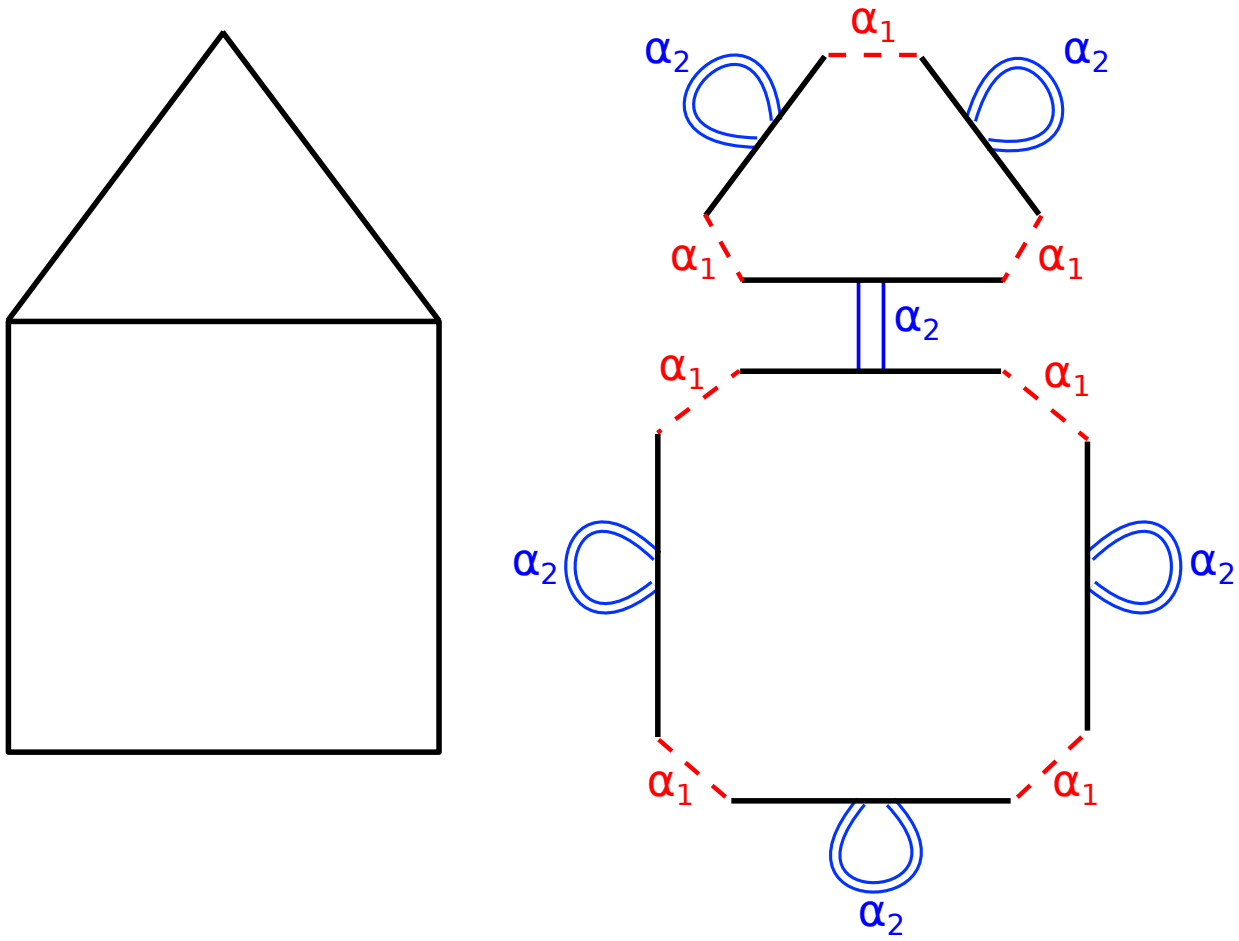

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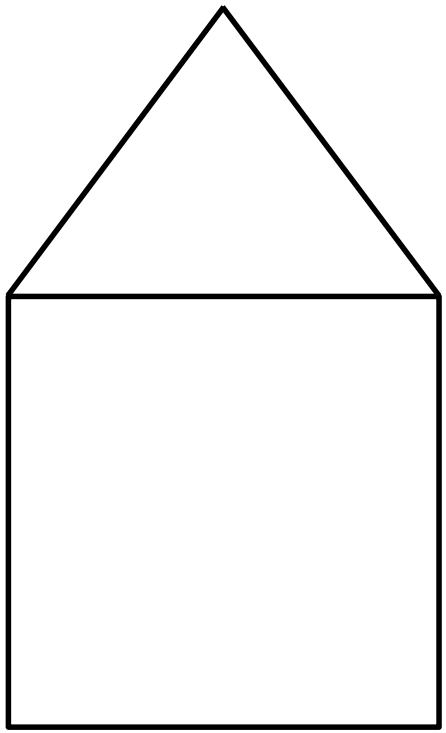
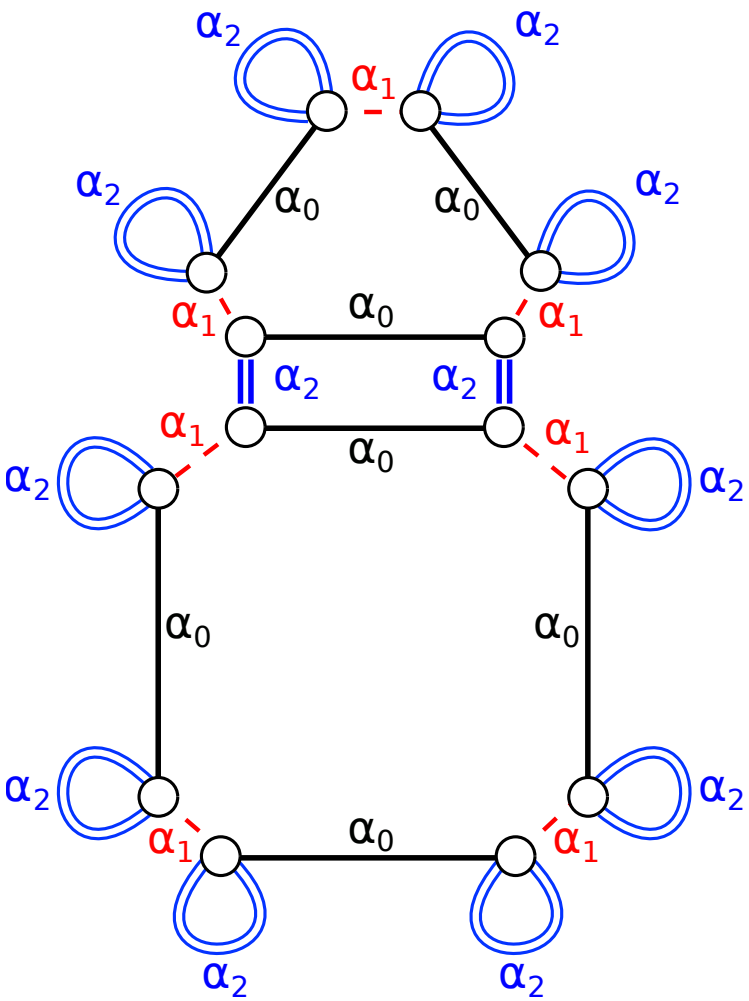
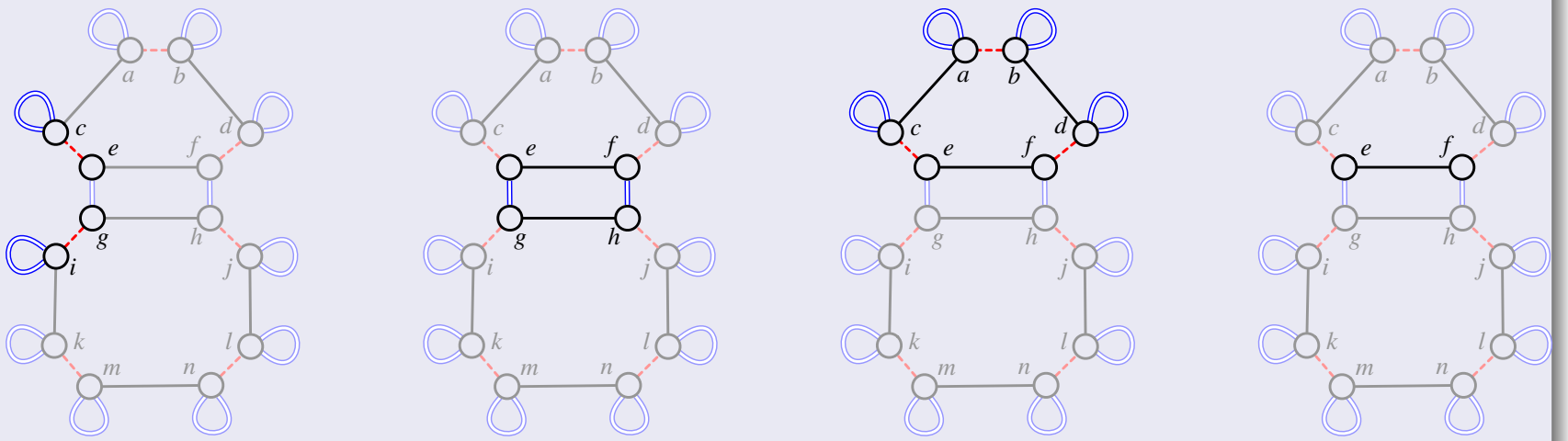




Cartes Généralisées

Orbite

Exemple de 2G-carte :



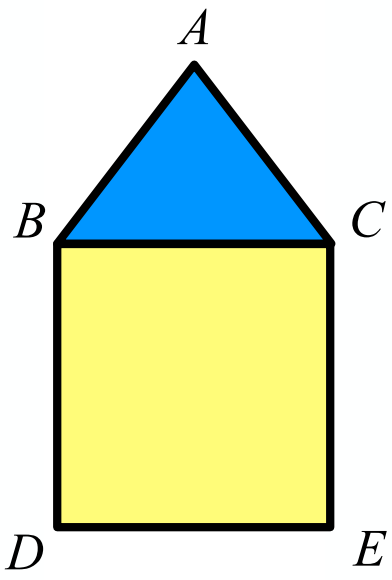
Sommet
$\langle \alpha_1, \alpha_2 \rangle(e)$

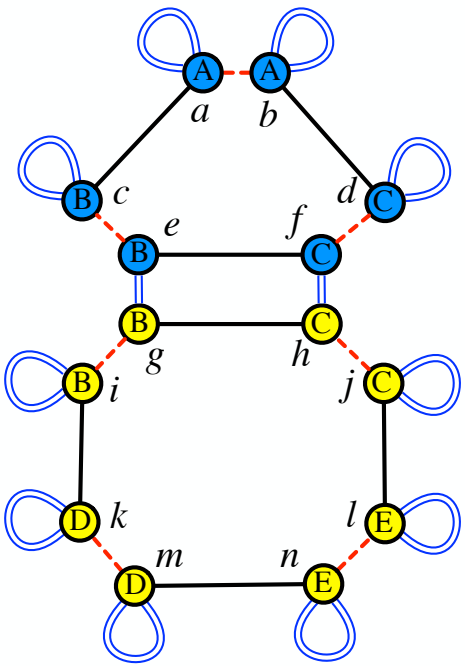
Arête
$\langle \alpha_0, \alpha_2 \rangle(e)$

Face
$\langle \alpha_0, \alpha_1 \rangle(e)$

Demi-arête
$\langle \alpha_0 \rangle(e)$

Objets





Topologie

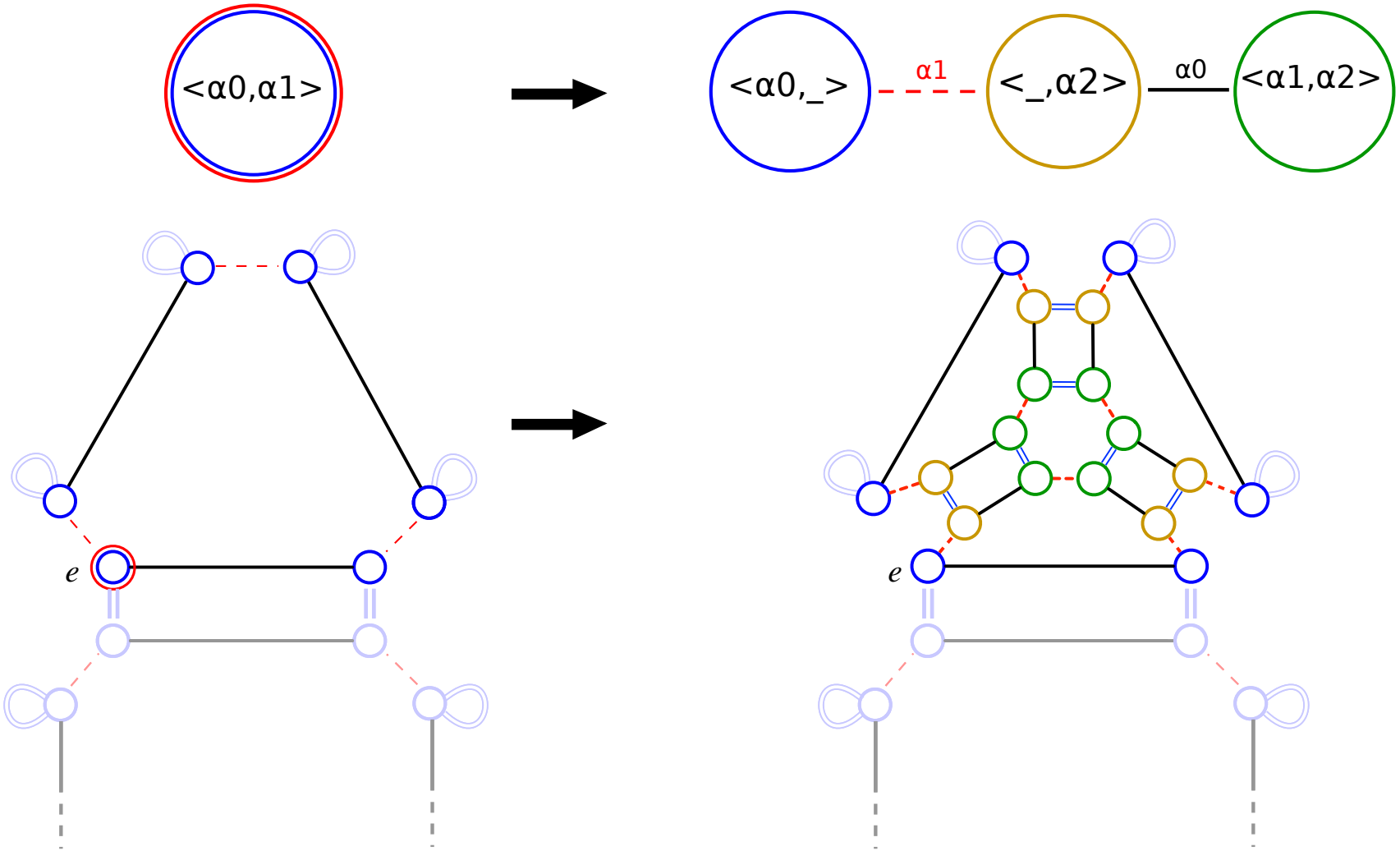
- Non orientation
- Arcs adjacents
- Cycle

Plongements

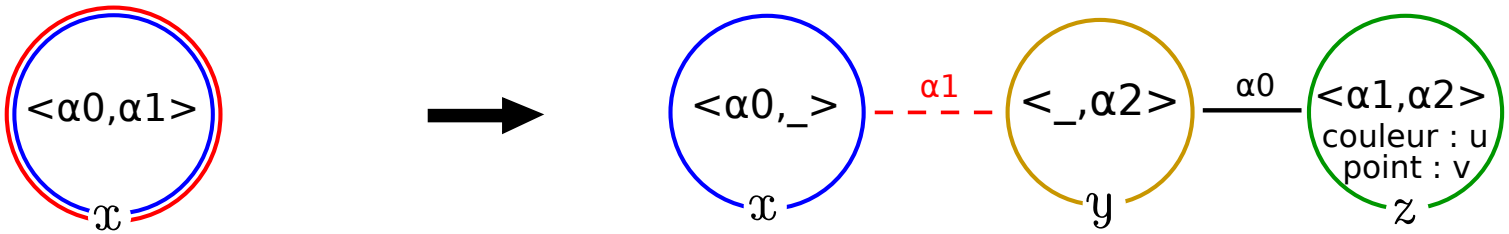
- Même valeur pour tous les nœuds de l'orbite.

Instanciation de la règle de triangulation barycentrique

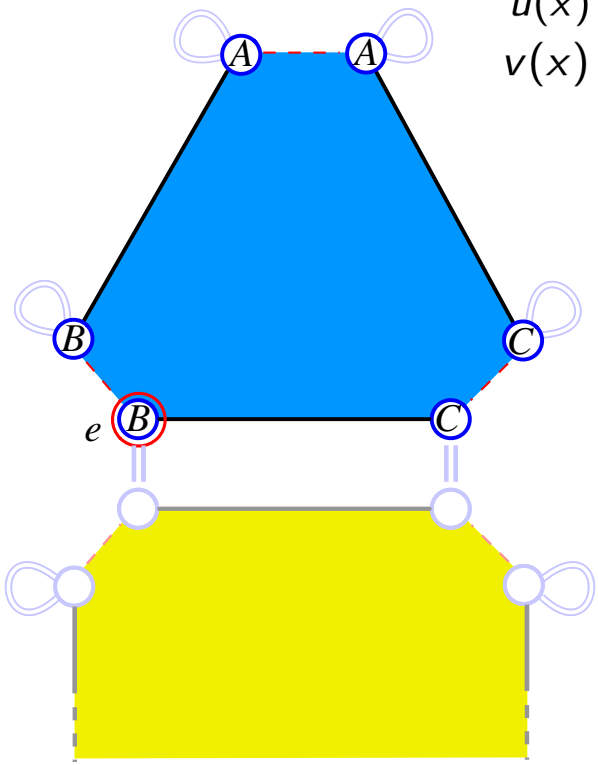
Création du barycentre de la face et liaison de ce nouveau point avec les sommets

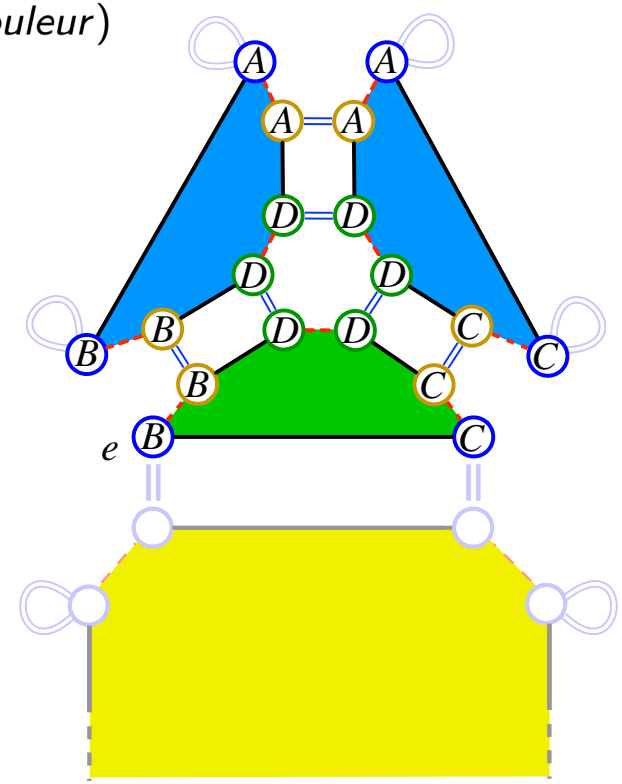


Expressions de plongement



$$u(x) = mix(x.couleur, x.\alpha_2.couleur)$$
$$v(x) = bary(point_{\langle 0,1 \rangle}(x))$$





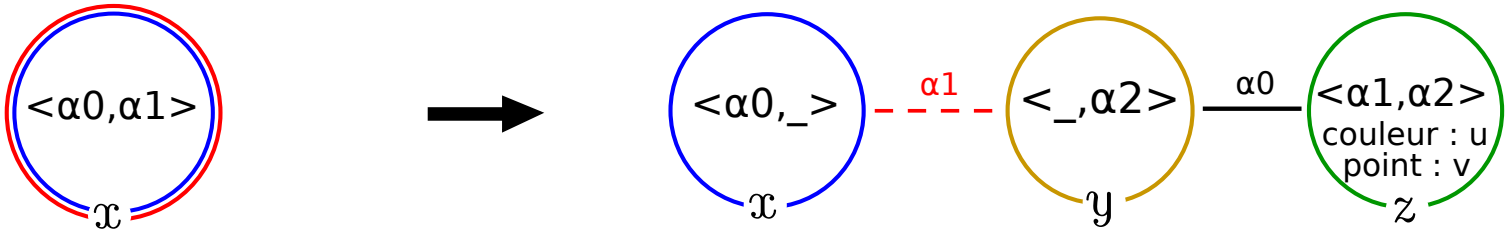
Détail du langage

Opération/instruction	Langage abstrait	Langage concret
Variable de nœud	x	`x`
Accès au plongement	$x.point$	`x.point`
Accès au voisin	$x.\alpha 1$	`x@1`
Collecte de plongements	$point_{\langle 0,1 \rangle}(x)$	$< 0, 1 >$_`point(x)`
...	...	...

Exemple

$mix(x.couleur, x.\alpha_2.couleur) \rightarrow$ `TCouleur::mix(x.couleur, x@2.couleur)`
$bary(point_{\langle 0,1 \rangle}(x)) \rightarrow$ `TPoint::bary(` $< 0, 1 >$_ `point(x))`

Condition



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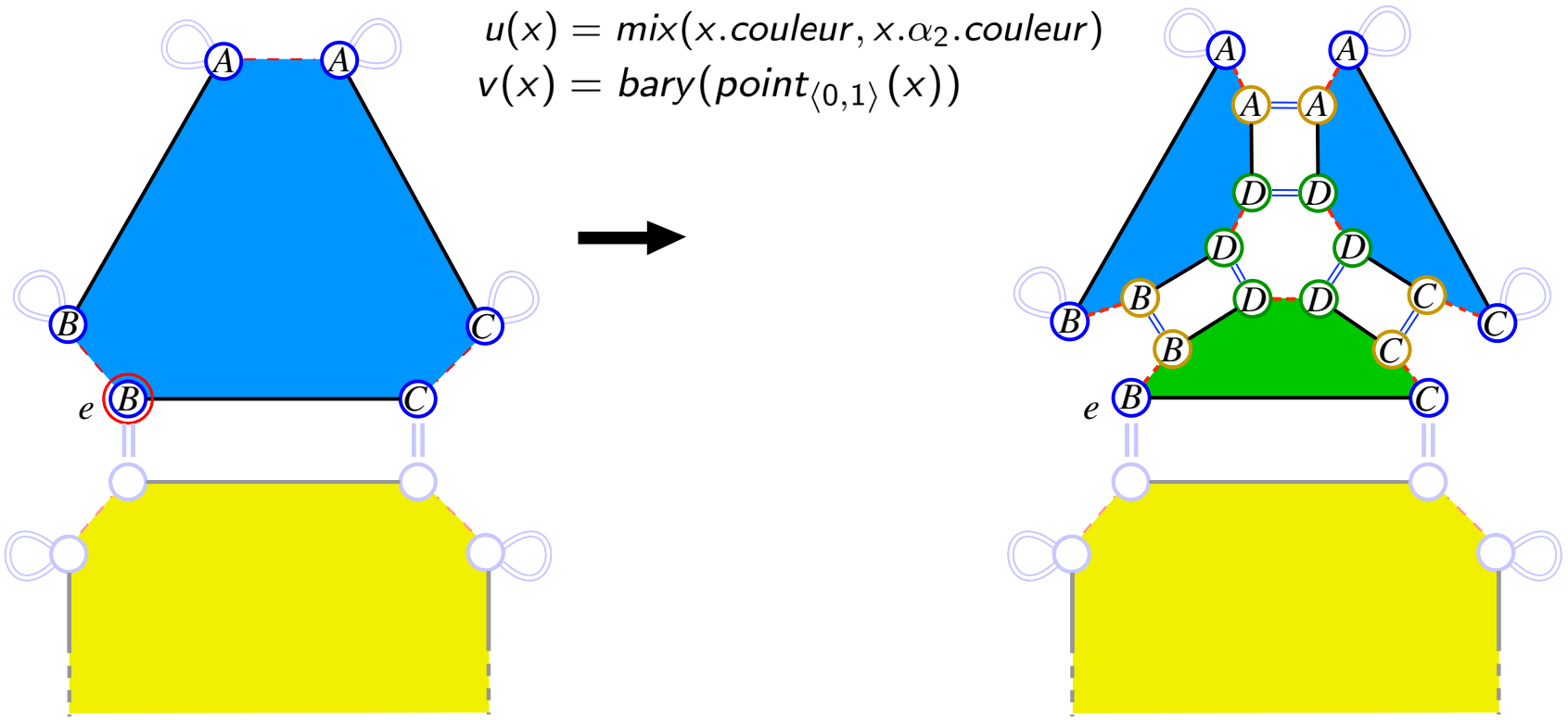

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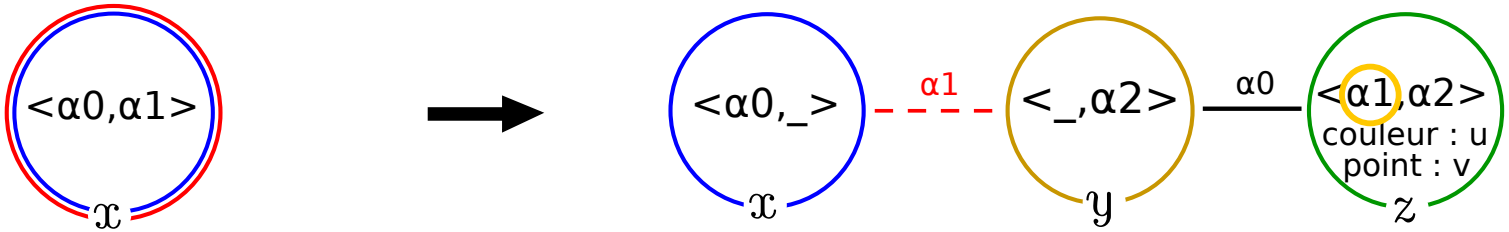

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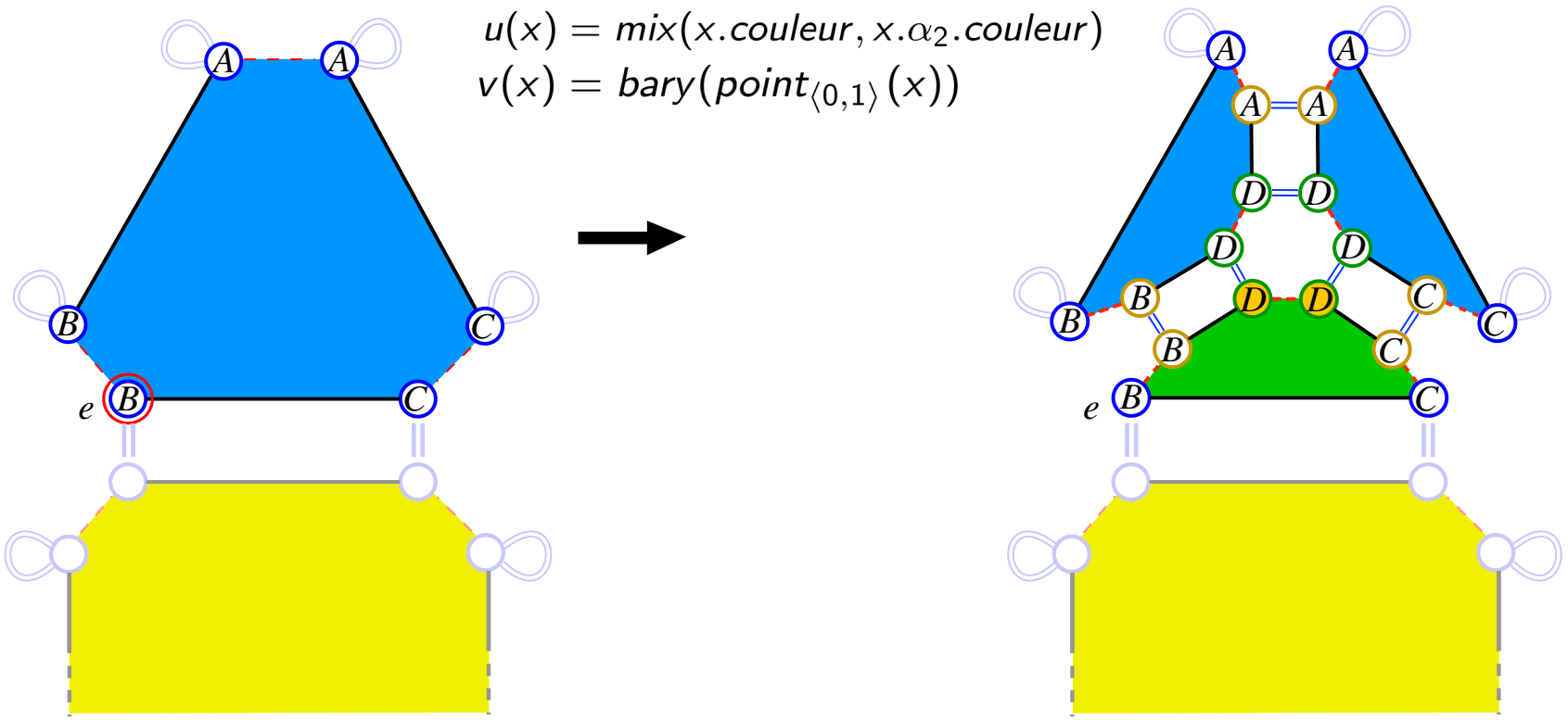

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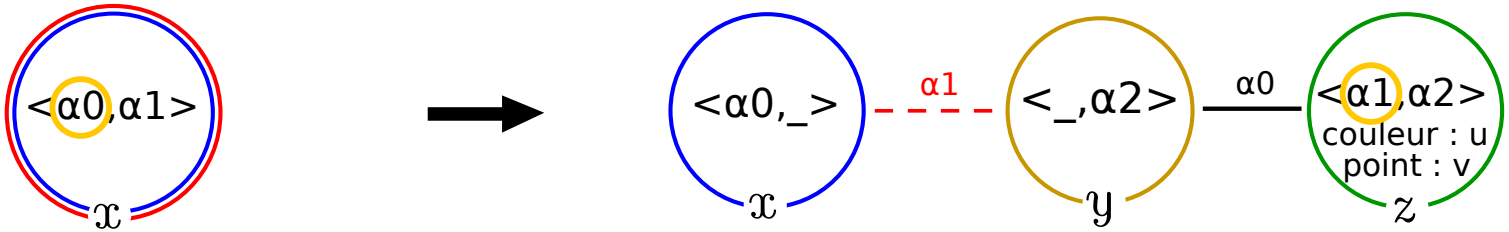

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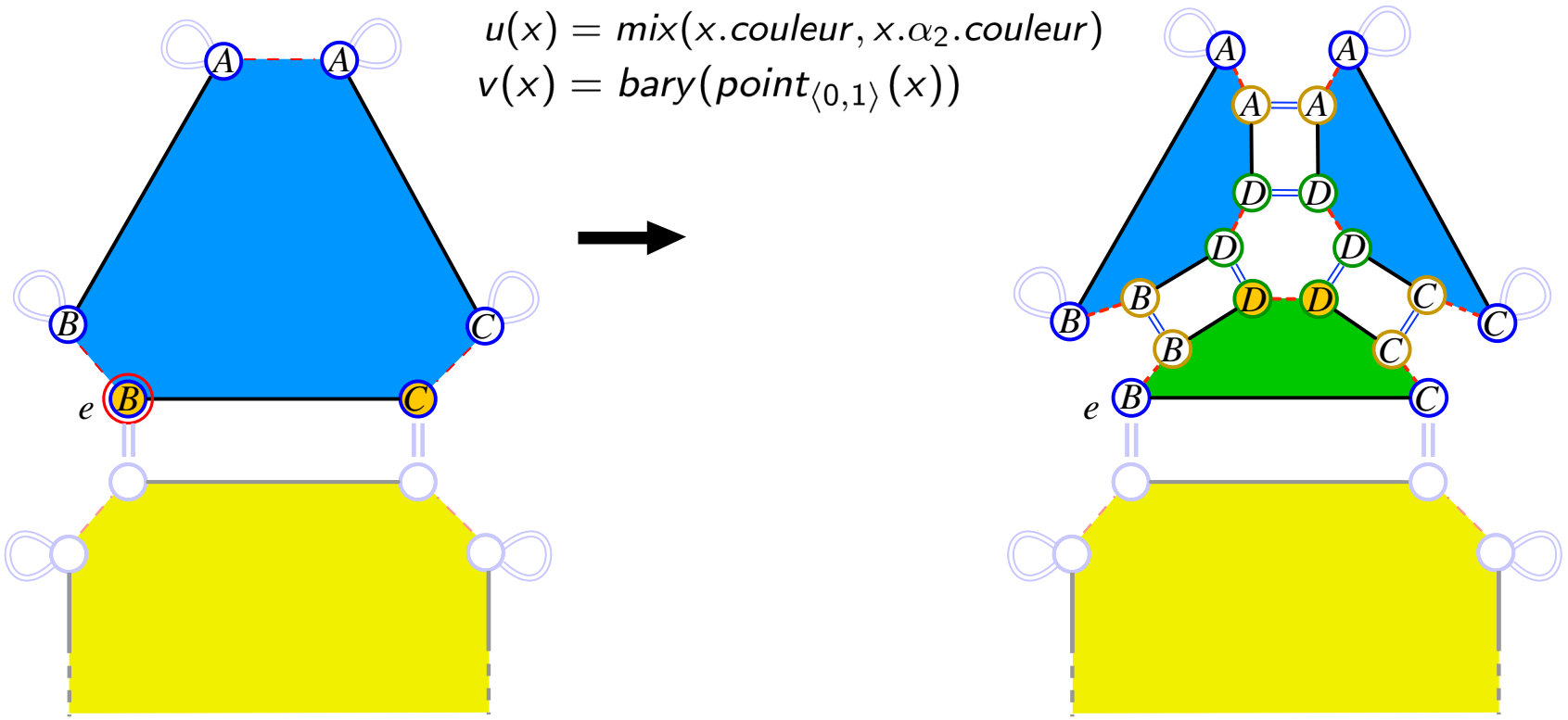

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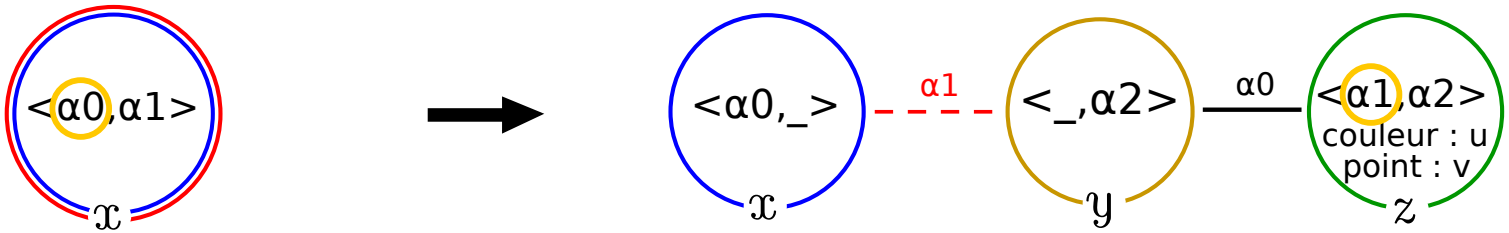

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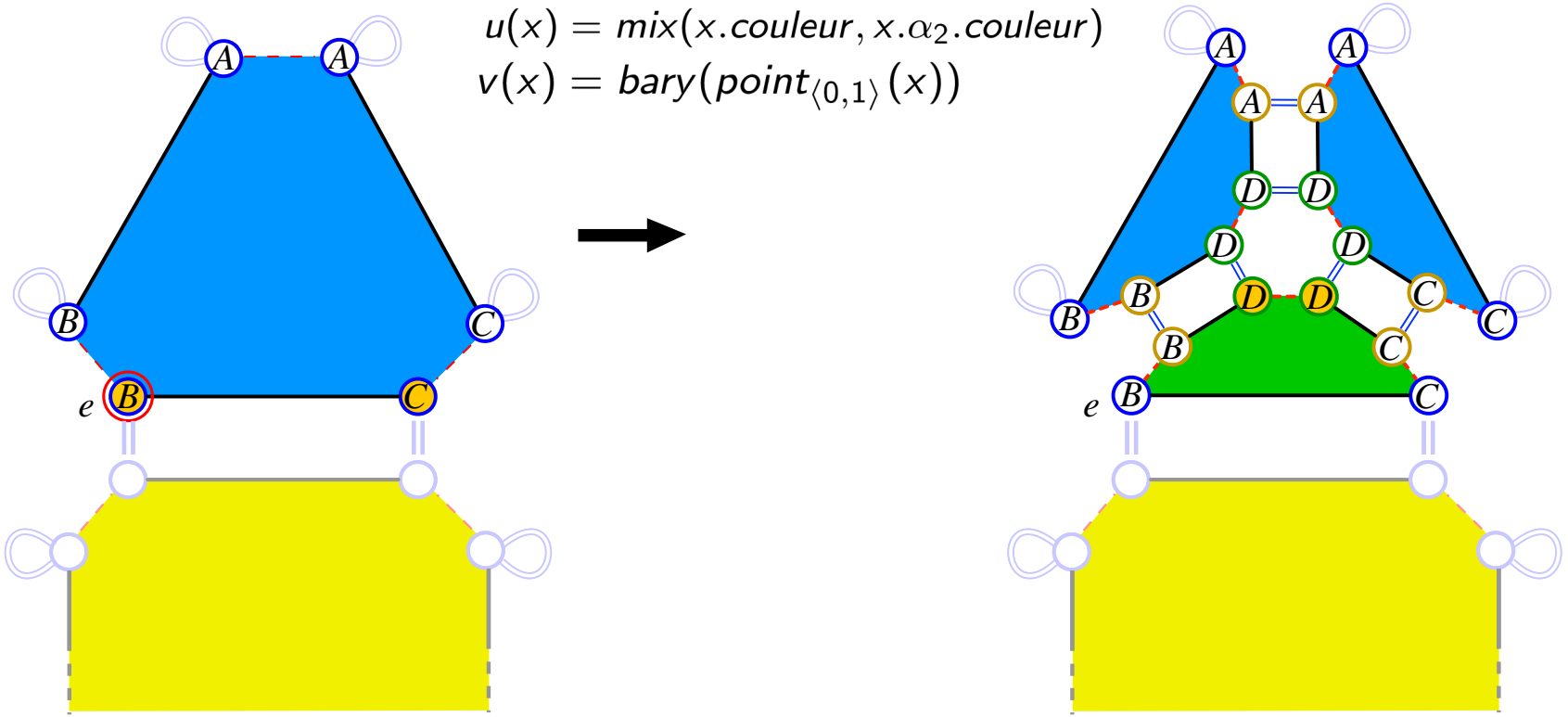
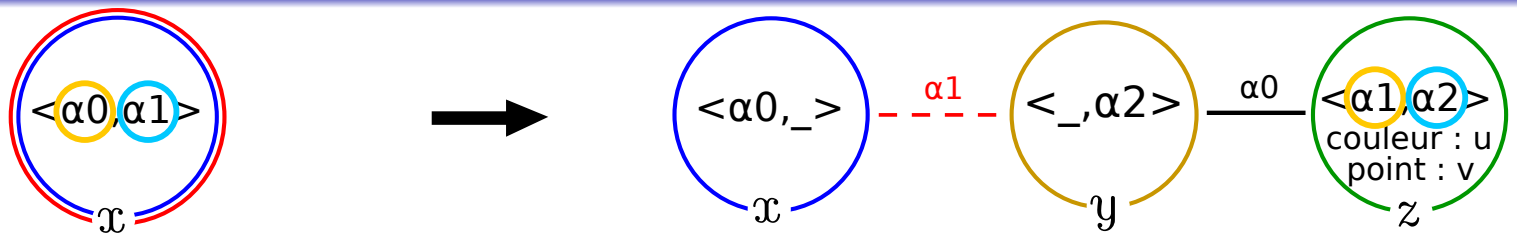


couleur : $v(x) \equiv_{\langle 0,1 \rangle} v(x.\alpha_0)$

Condition



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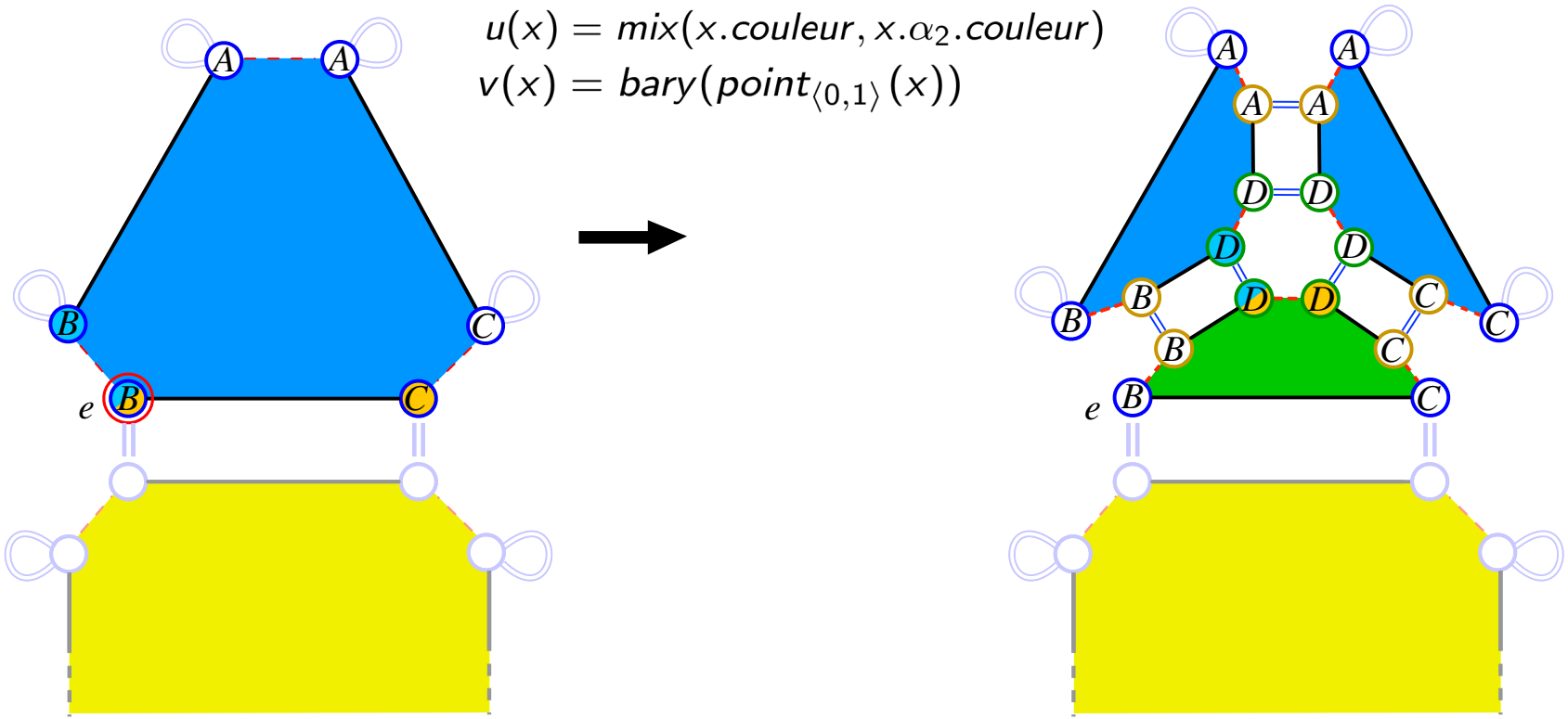
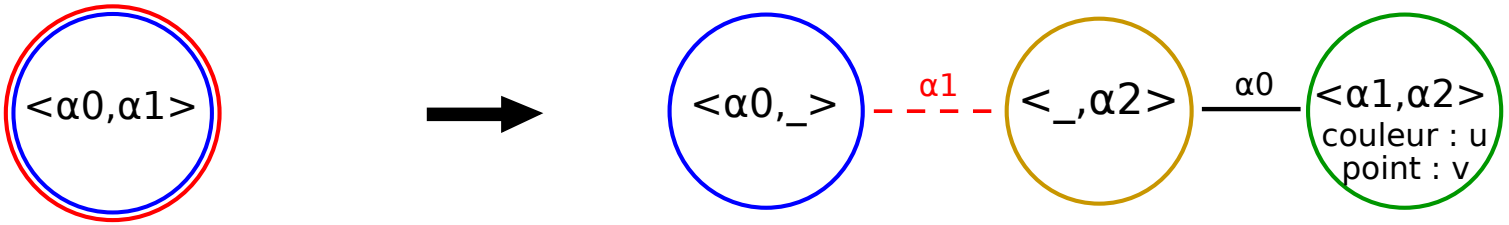


couleur : $v(x) \equiv_{\langle 0,1\rangle} v(x.\alpha_0)$
point : $v(x) \equiv_{\langle 1,2\rangle} v(x.\alpha_0)$ et $v(x) \equiv_{\langle 1,2\rangle} v(x.\alpha_1)$

Condition



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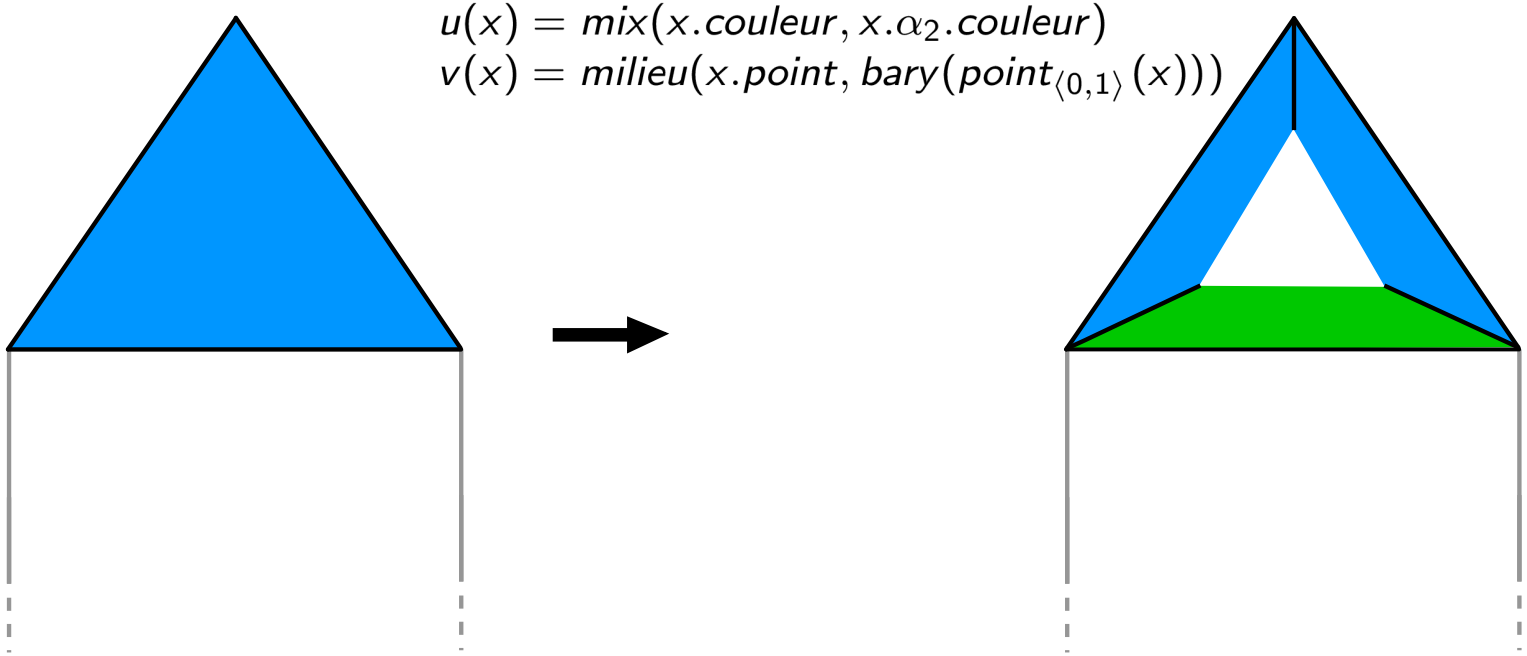

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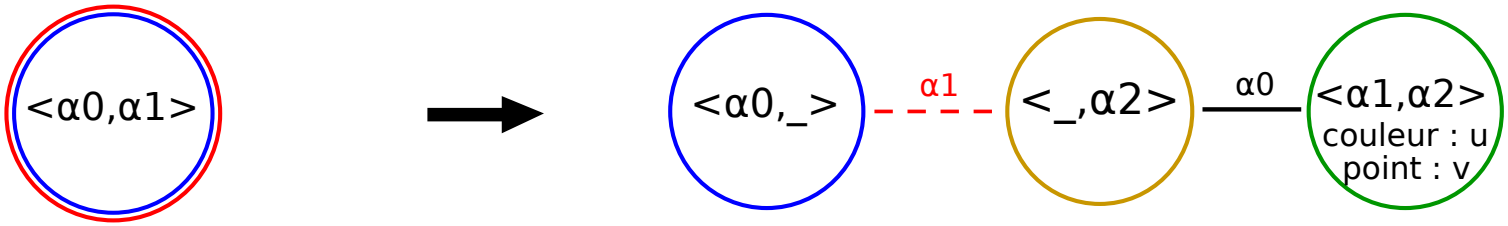

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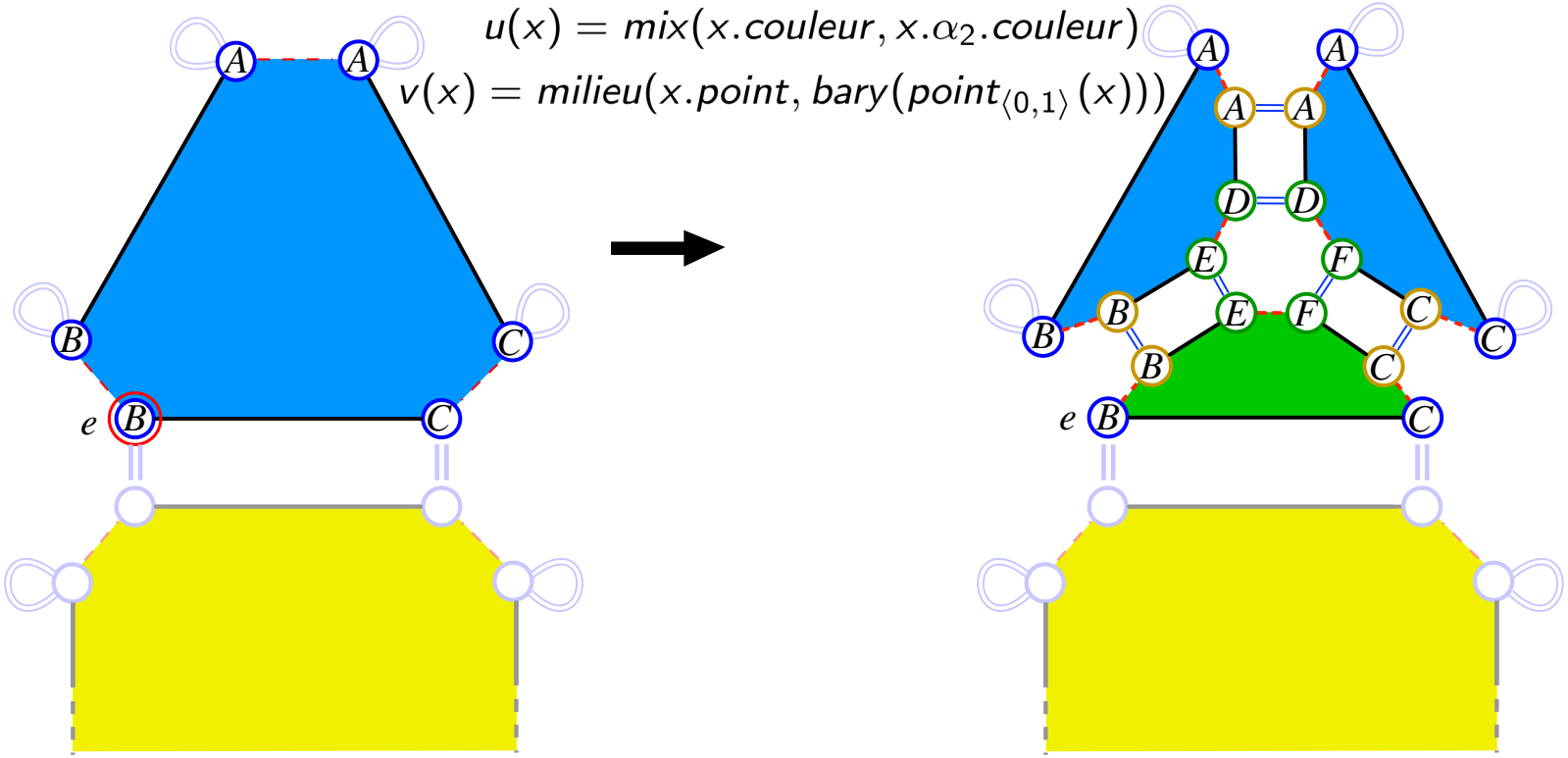

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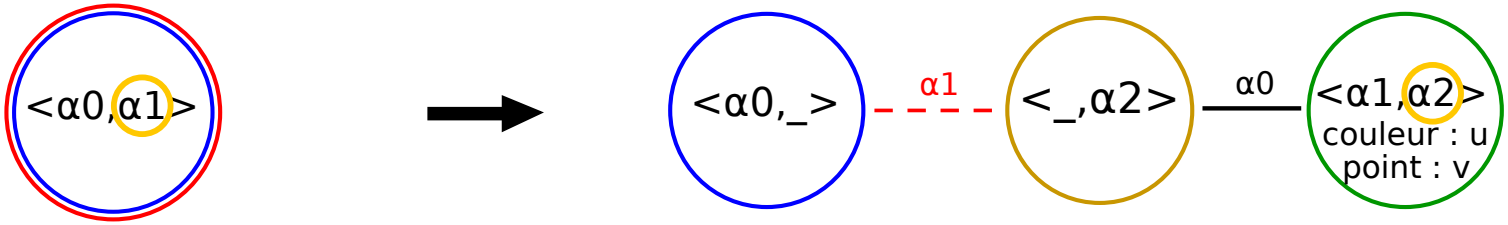

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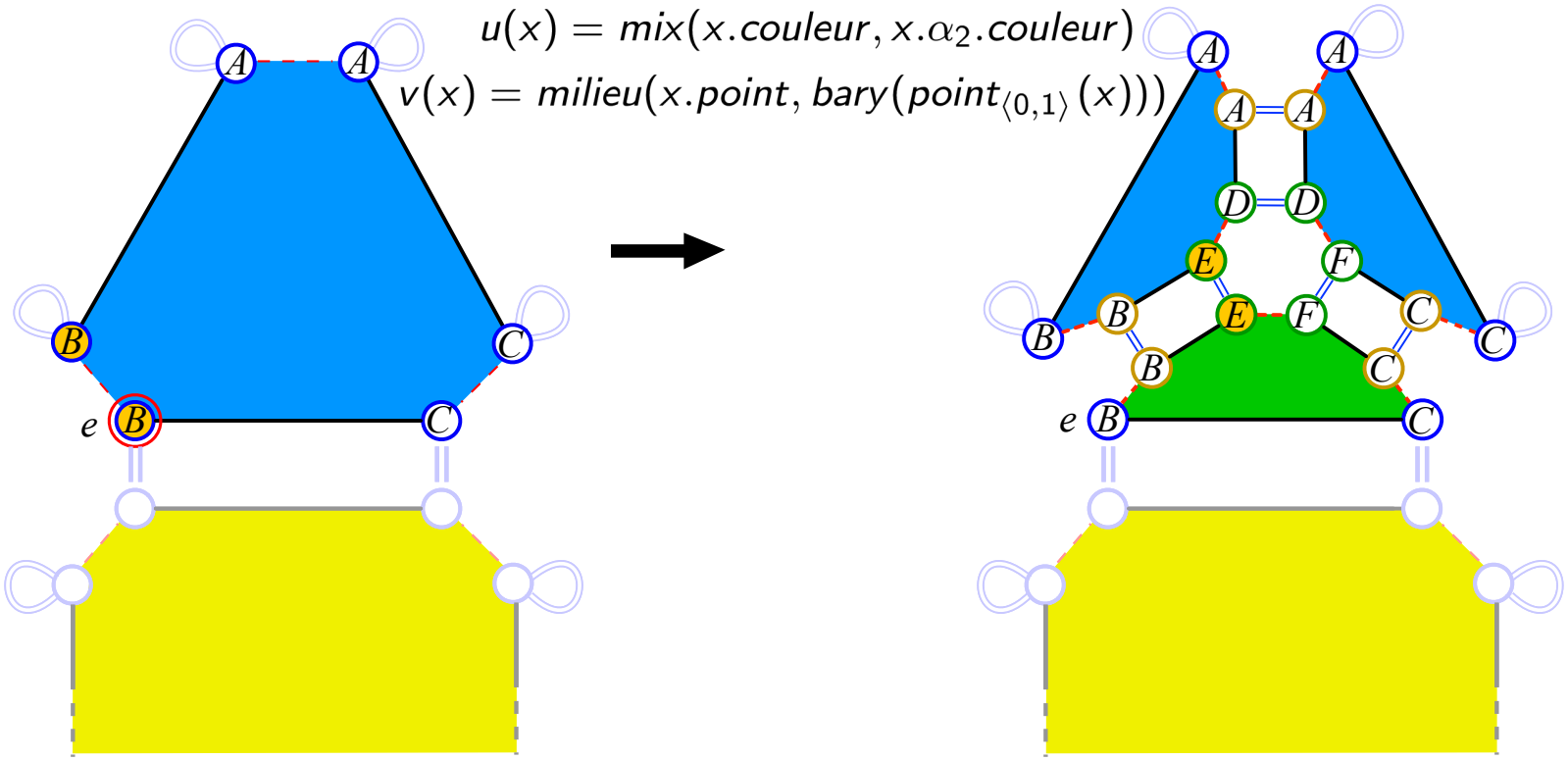
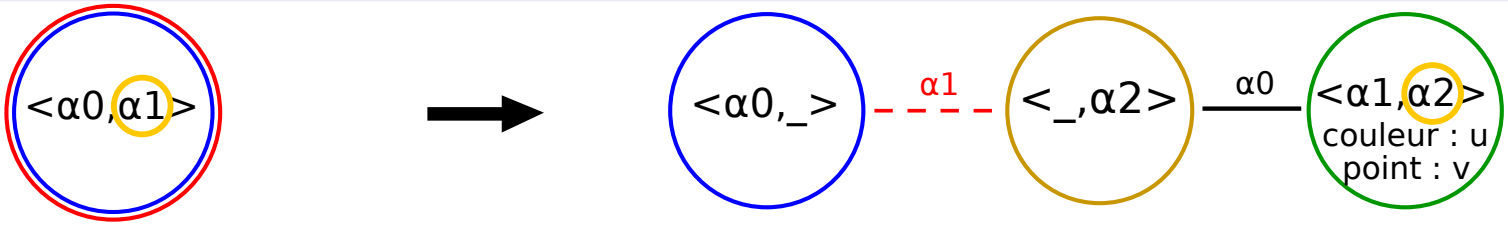


$$v(x) \equiv_{\langle 1,2 \rangle} v(x.\alpha_1)$$

Condition



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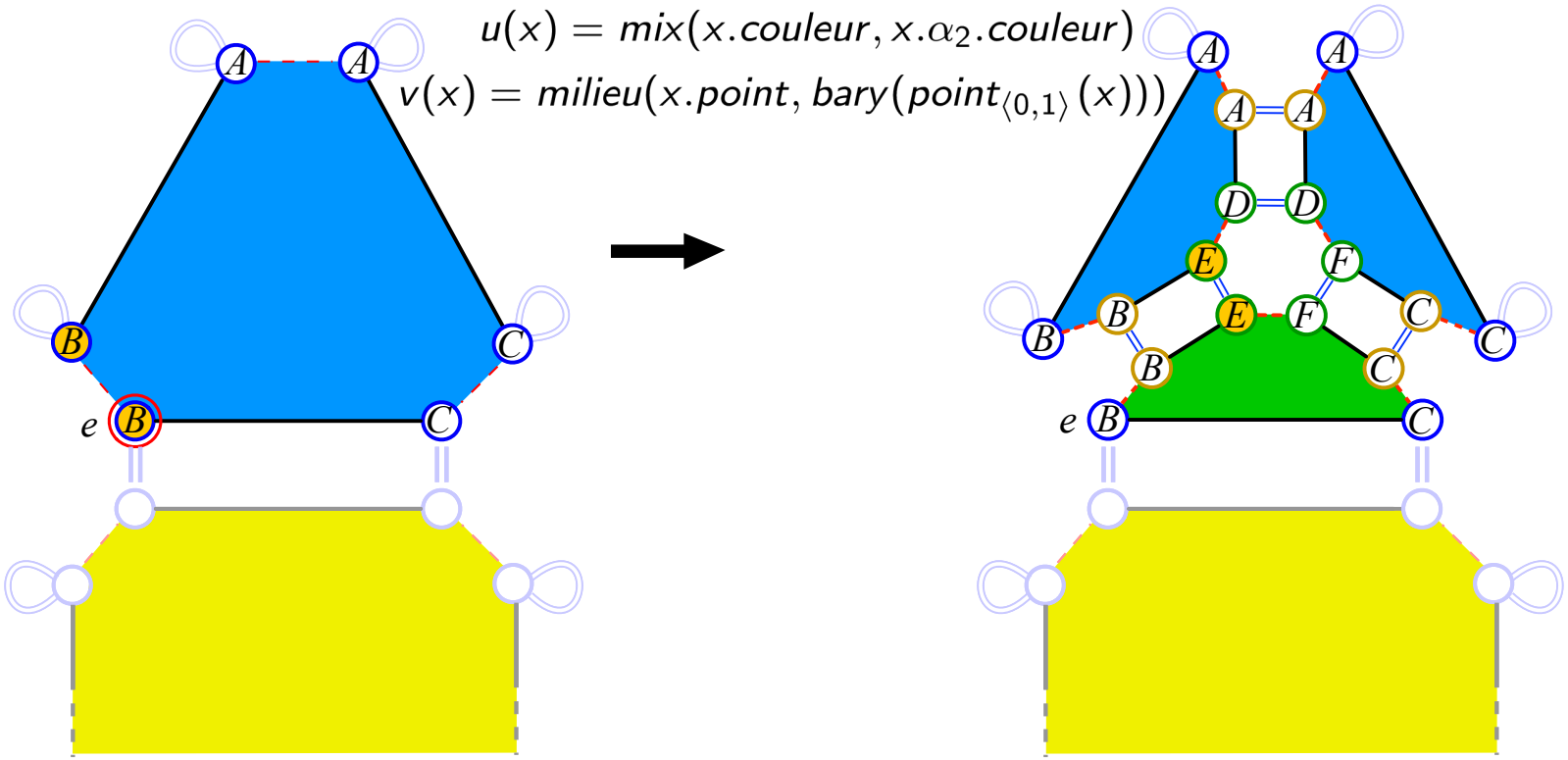
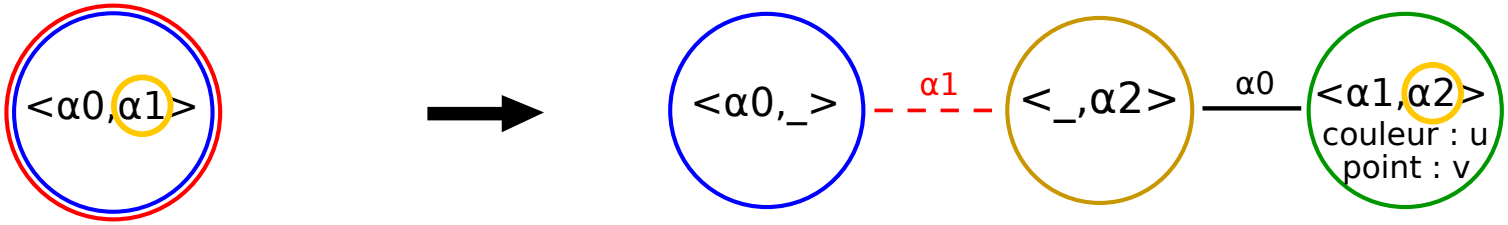


$$milieu(x.point, bary(point_{\langle 0,1\rangle}(x))) \equiv_{\langle 1,2\rangle} milieu(x.\alpha_1.point, bary(point_{\langle 0,1\rangle}(x.\alpha_1)))$$

Condition



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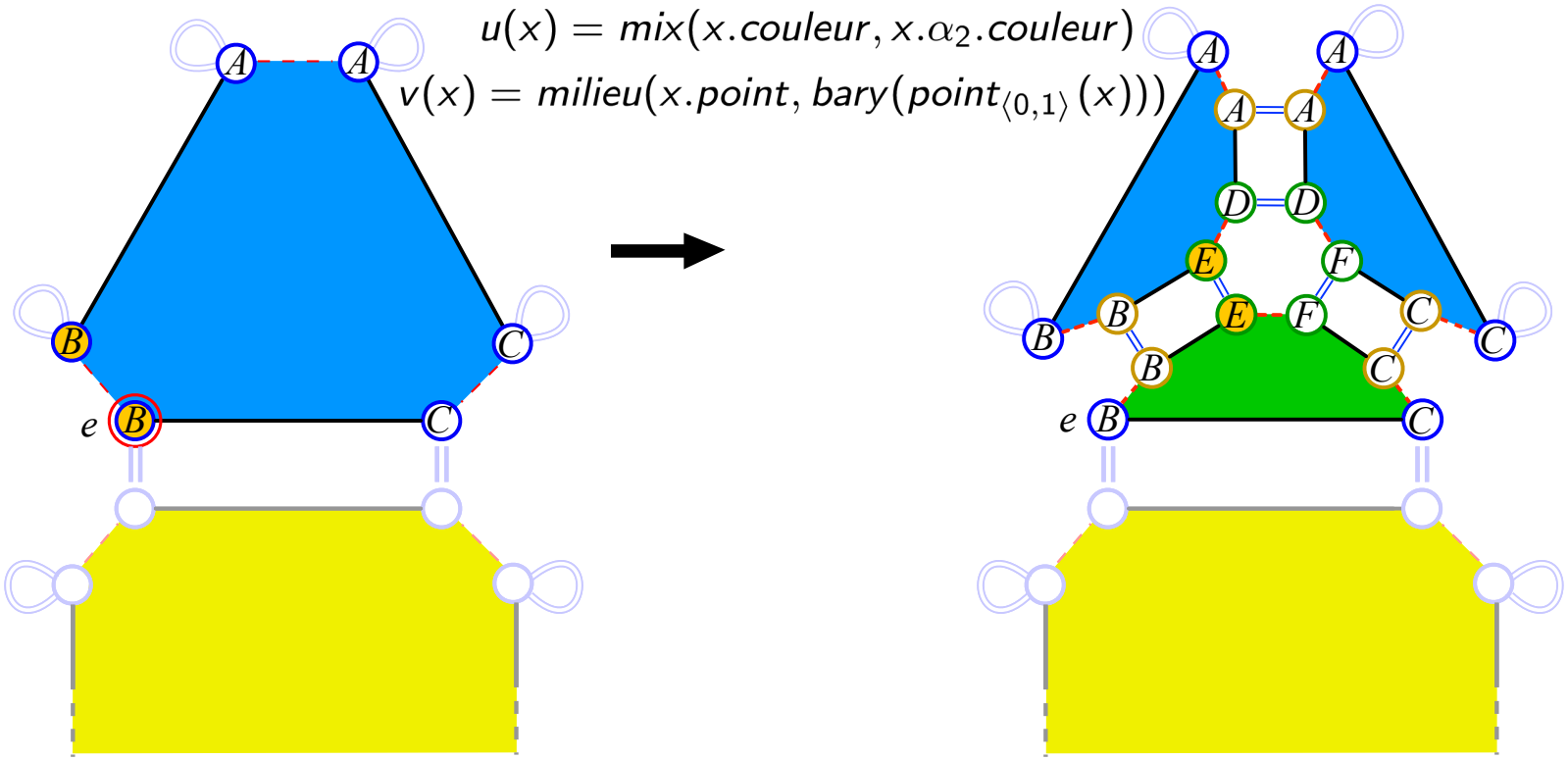

$x.point \equiv_{\langle 1,2 \rangle} x.\alpha_1.point$ et $bary(point_{\langle 0,1 \rangle}(x)) \equiv_{\langle 1,2 \rangle} bary(point_{\langle 0,1 \rangle}(x.\alpha_1))$

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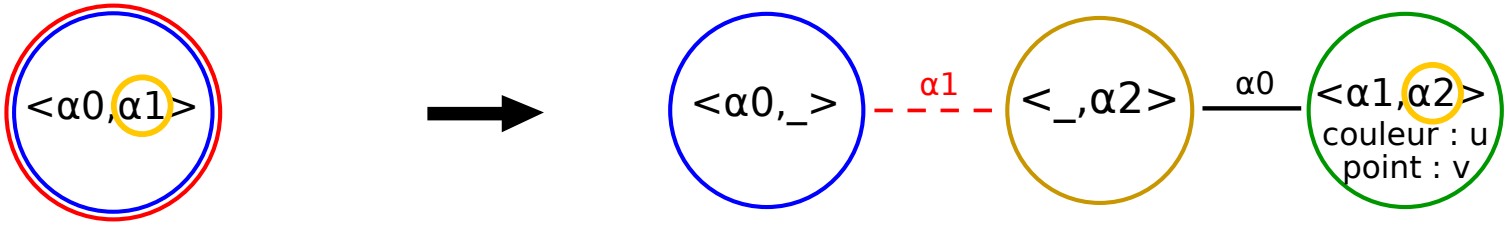

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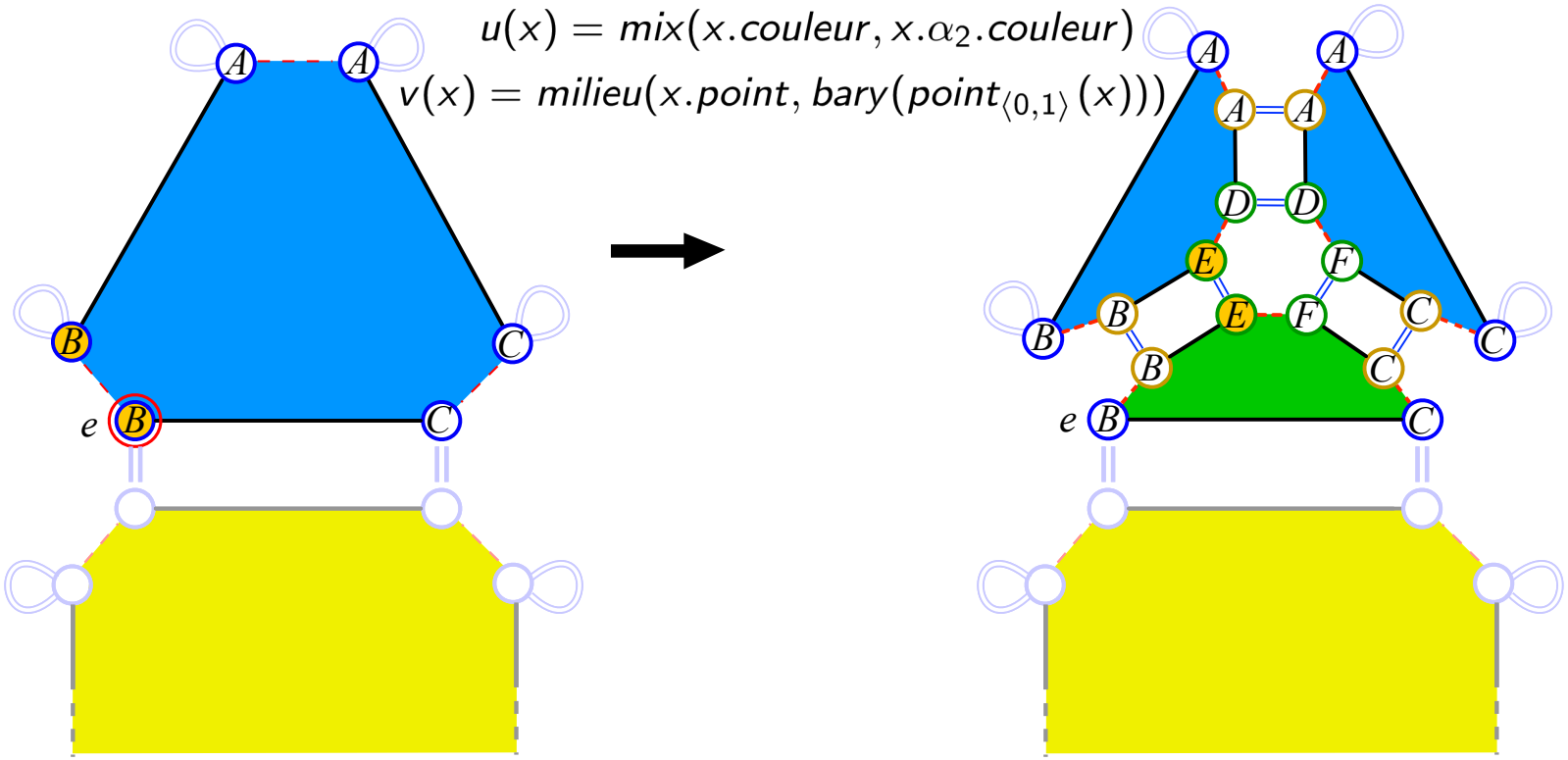

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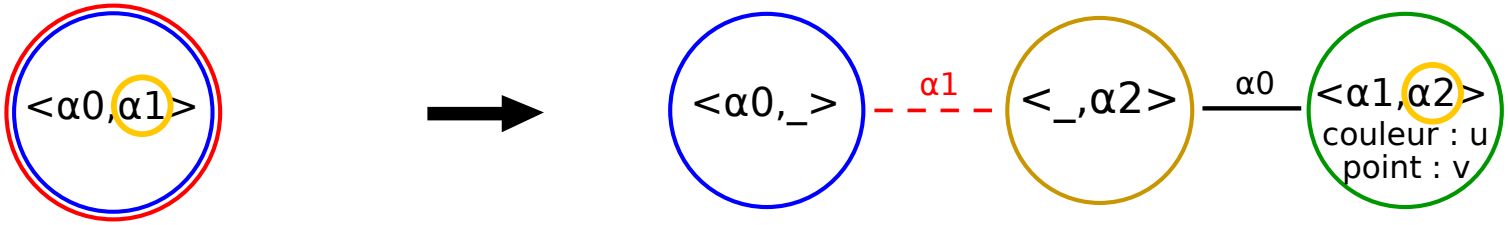

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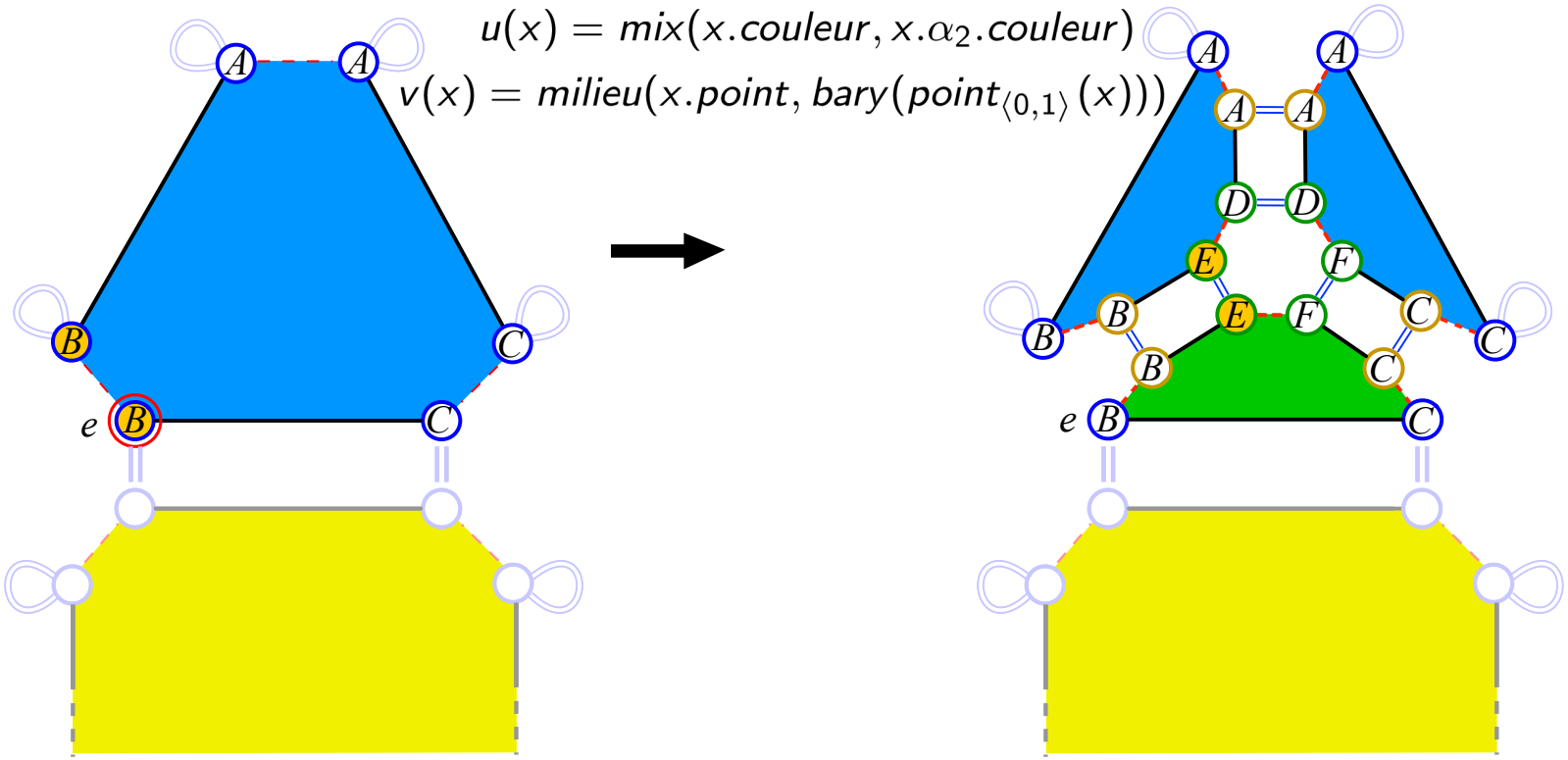

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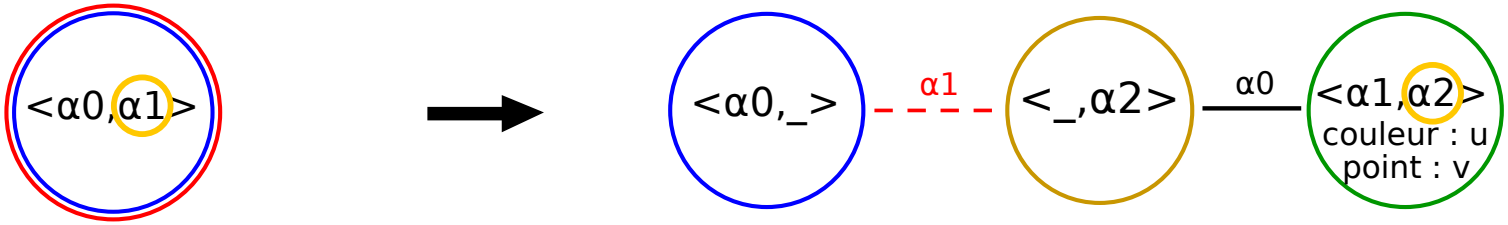

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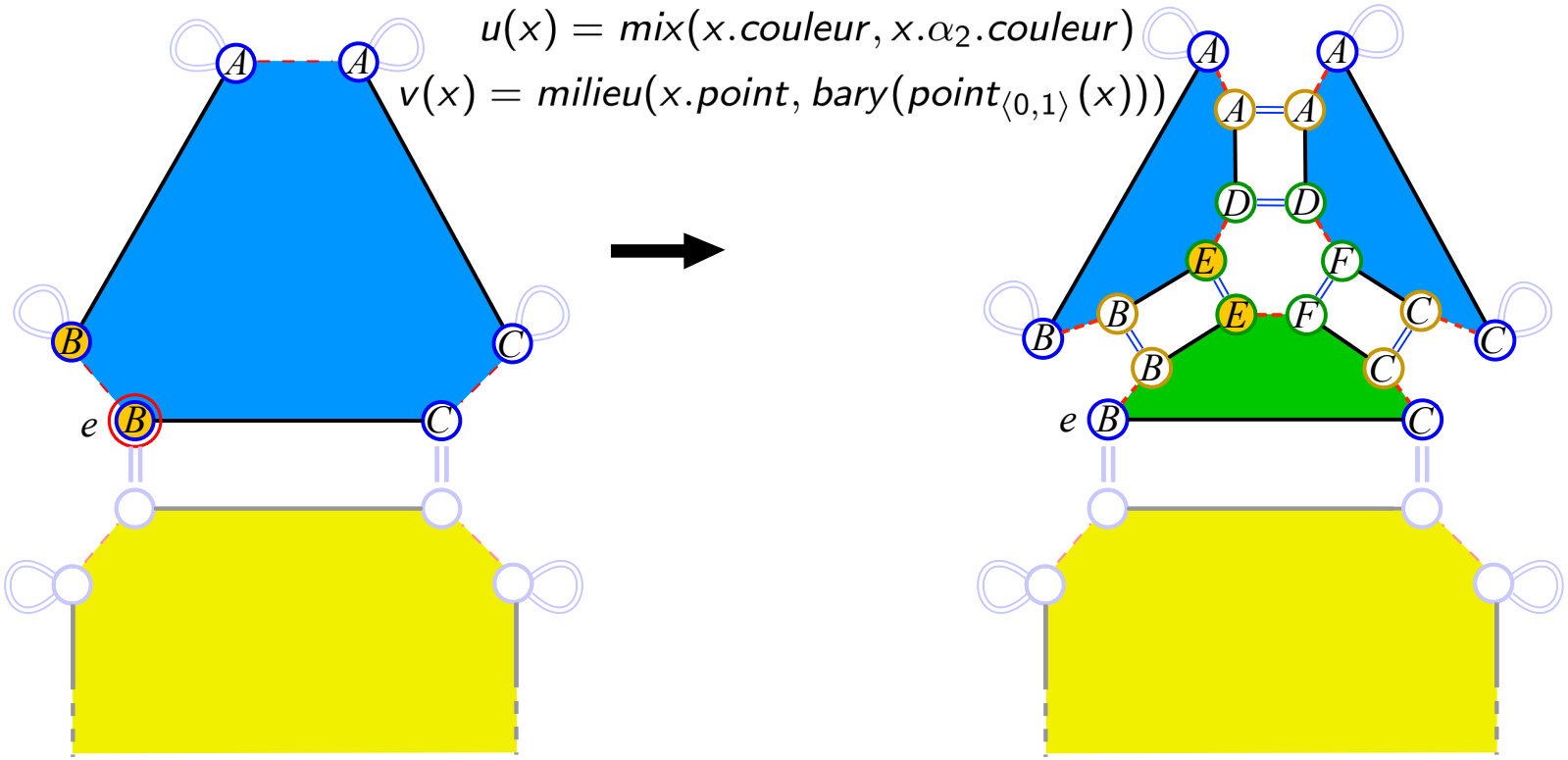

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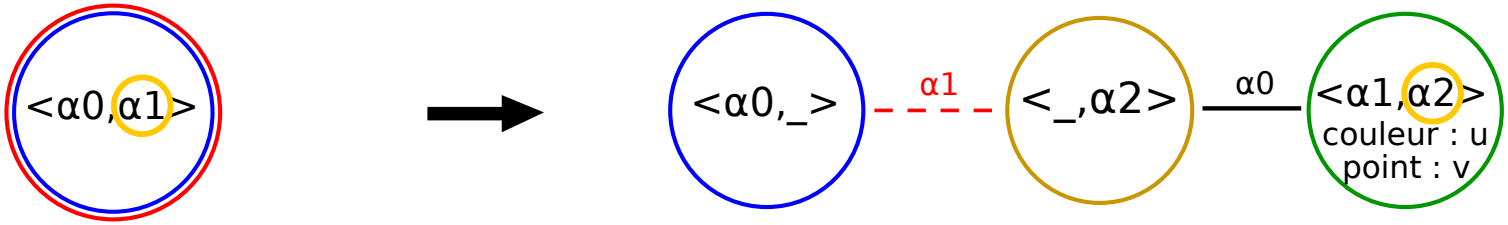

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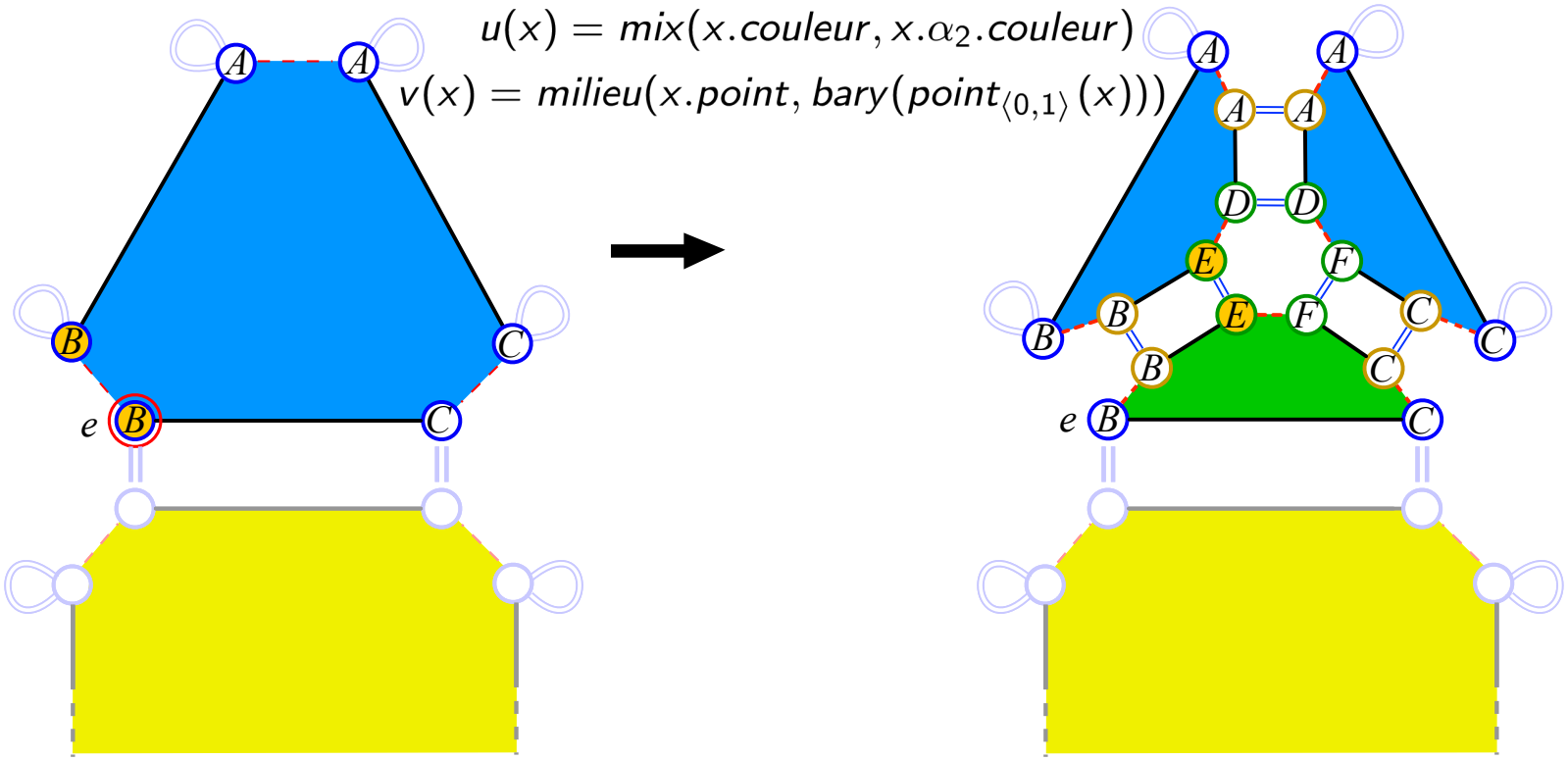

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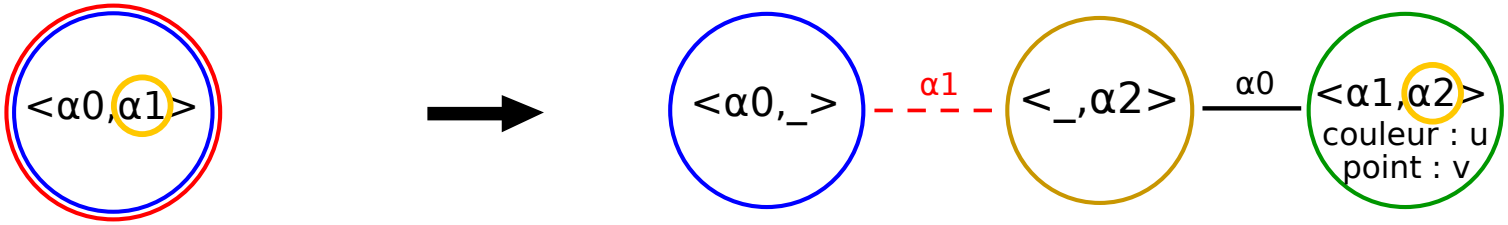

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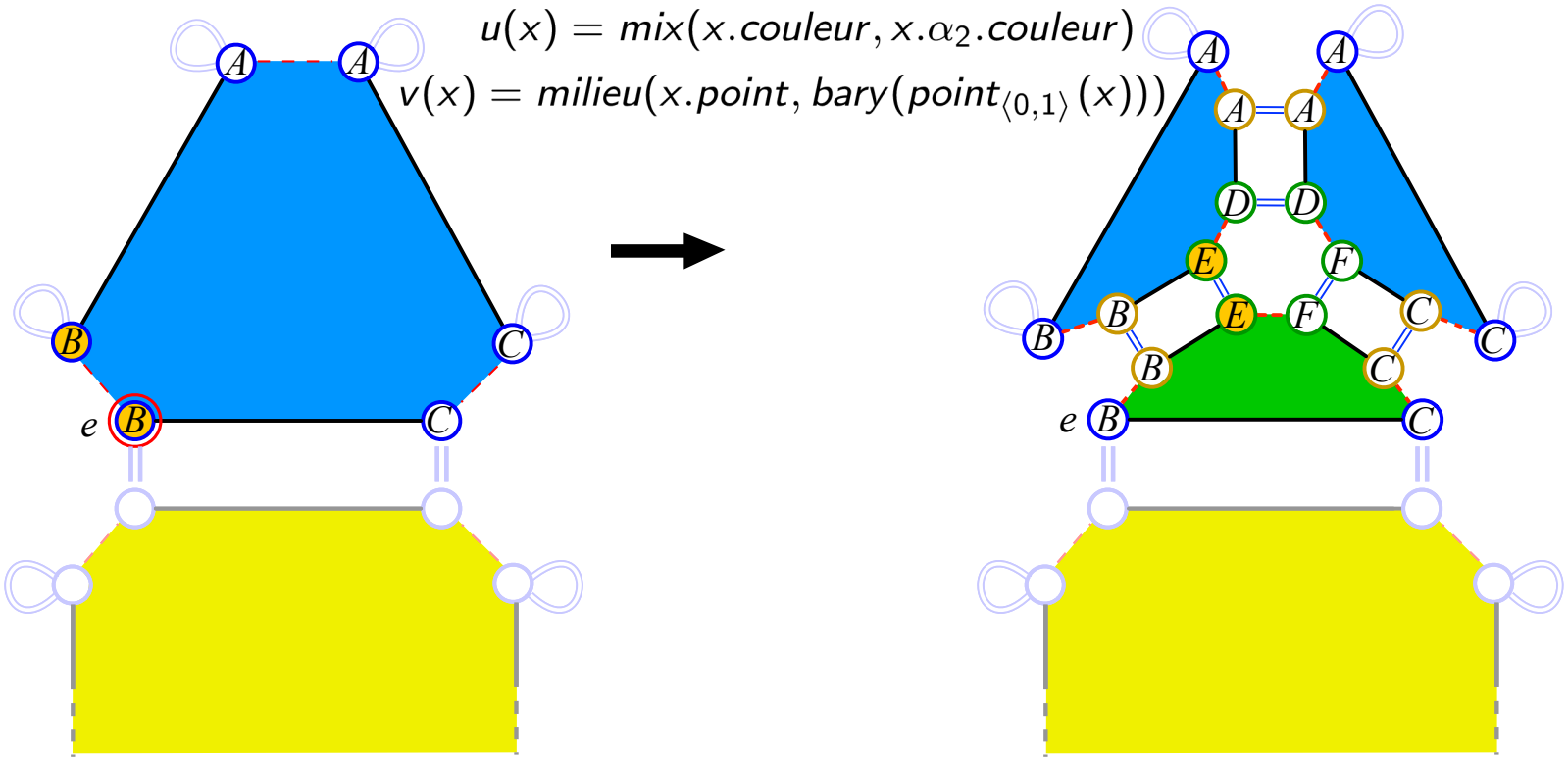
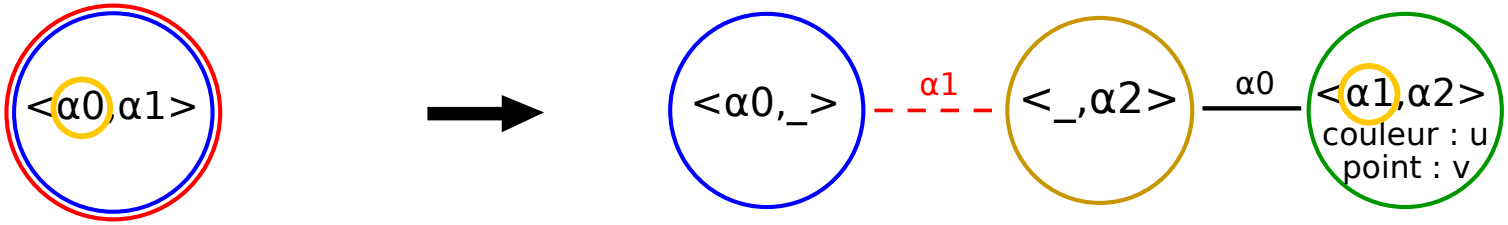
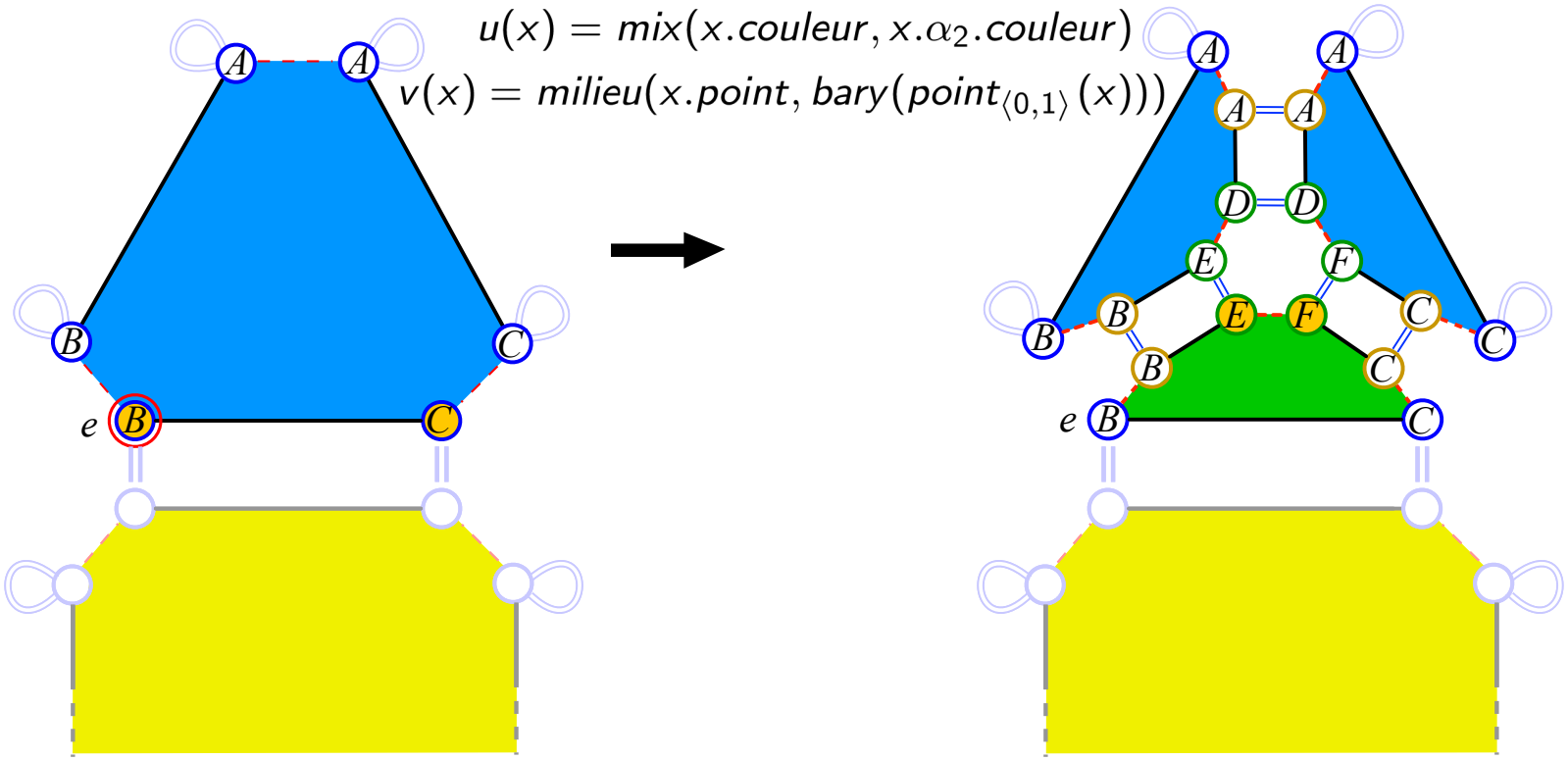


$x \equiv_{\langle 1,2 \rangle} x.\alpha_1$ et $x \equiv_{\langle 0,1 \rangle} x.\alpha_1$

Condition

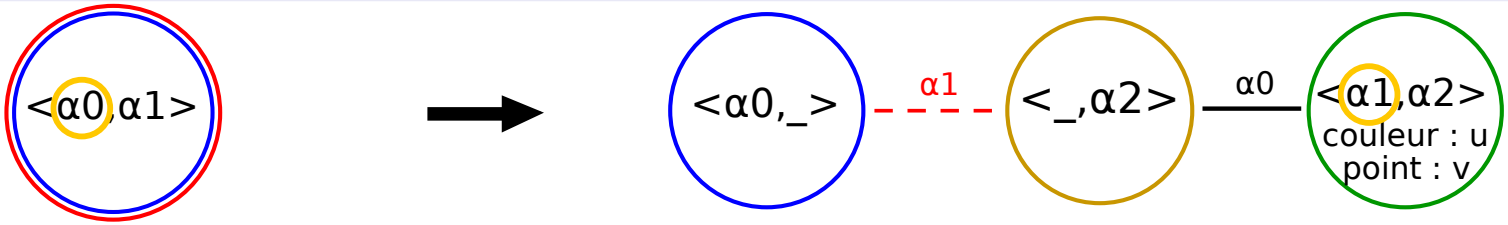


$$u(x) = mix(x.couleur, x.\alpha_2.couleur)$$
$$v(x) = milieu(x.point, bary(point_{\langle 0,1 \rangle}(x)))$$

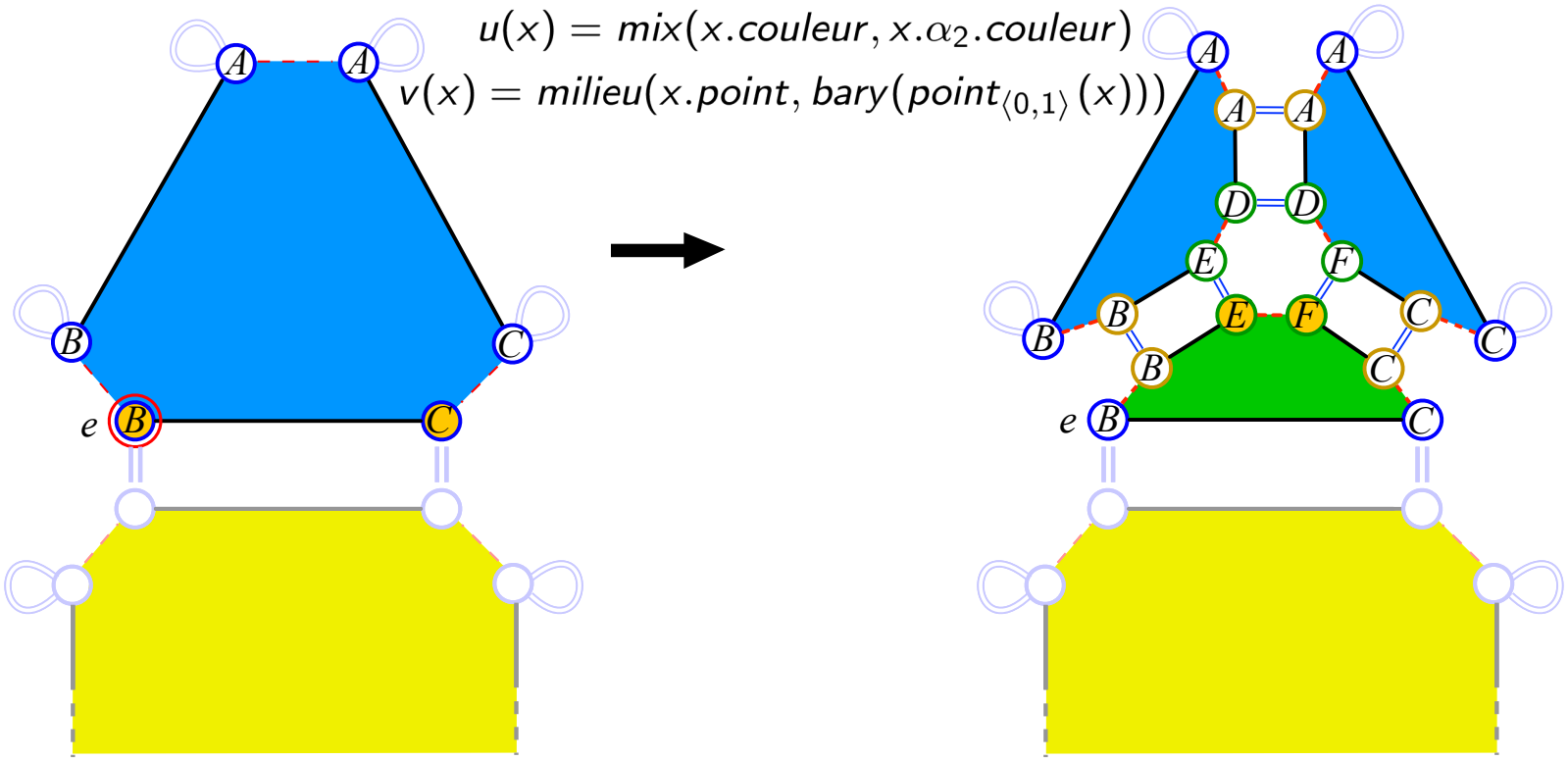


$$v(x) \equiv_{\langle 1,2 \rangle} v(x.\alpha_0)$$

Condition

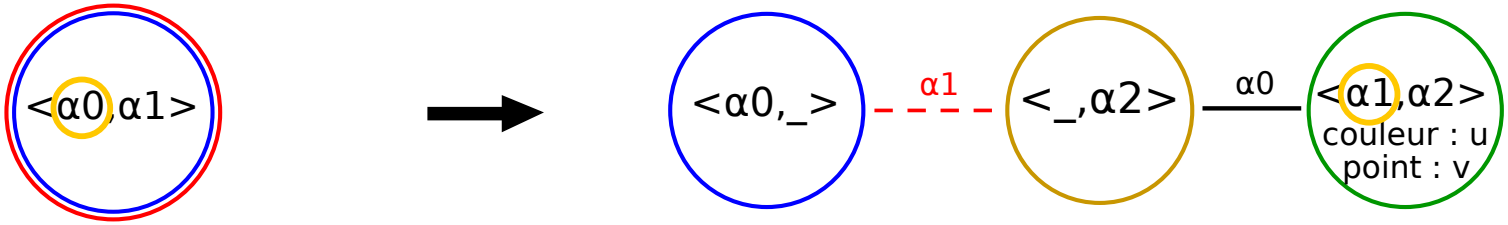


$$u(x) = mix(x.couleur, x.\alpha_2.couleur)$$
$$v(x) = milieu(x.point, bary(point_{\langle 0,1 \rangle}(x)))$$



$$milieu(x.point, bary(point_{\langle 0,1 \rangle}(x))) \equiv_{\langle 1,2 \rangle} milieu(x.\alpha_0.point, bary(point_{\langle 0,1 \rangle}(x.\alpha_0)))$$

Condition



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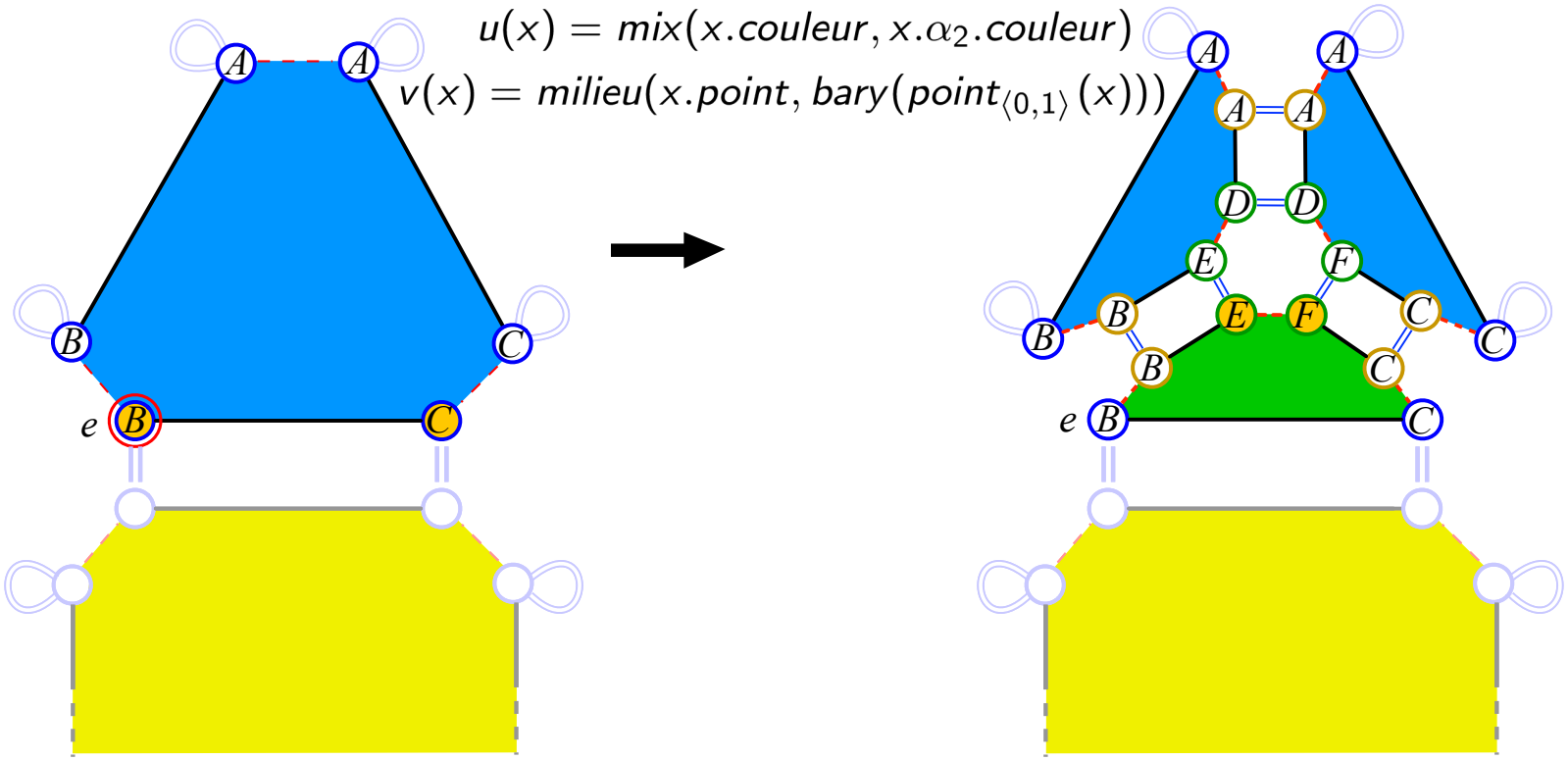
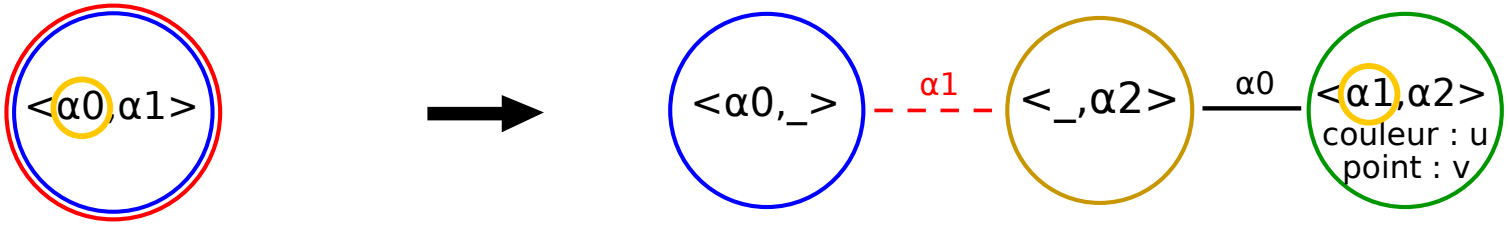


$x.point \equiv_{\langle 1,2 \rangle} x.\alpha_0.point$ et $bary(point_{\langle 0,1 \rangle}(x)) \equiv_{\langle 1,2 \rangle} bary(point_{\langle 0,1 \rangle}(x.\alpha_0))$

Condition



$$u(x) = mix(x.couleur, x.\alpha_2.couleur)$$
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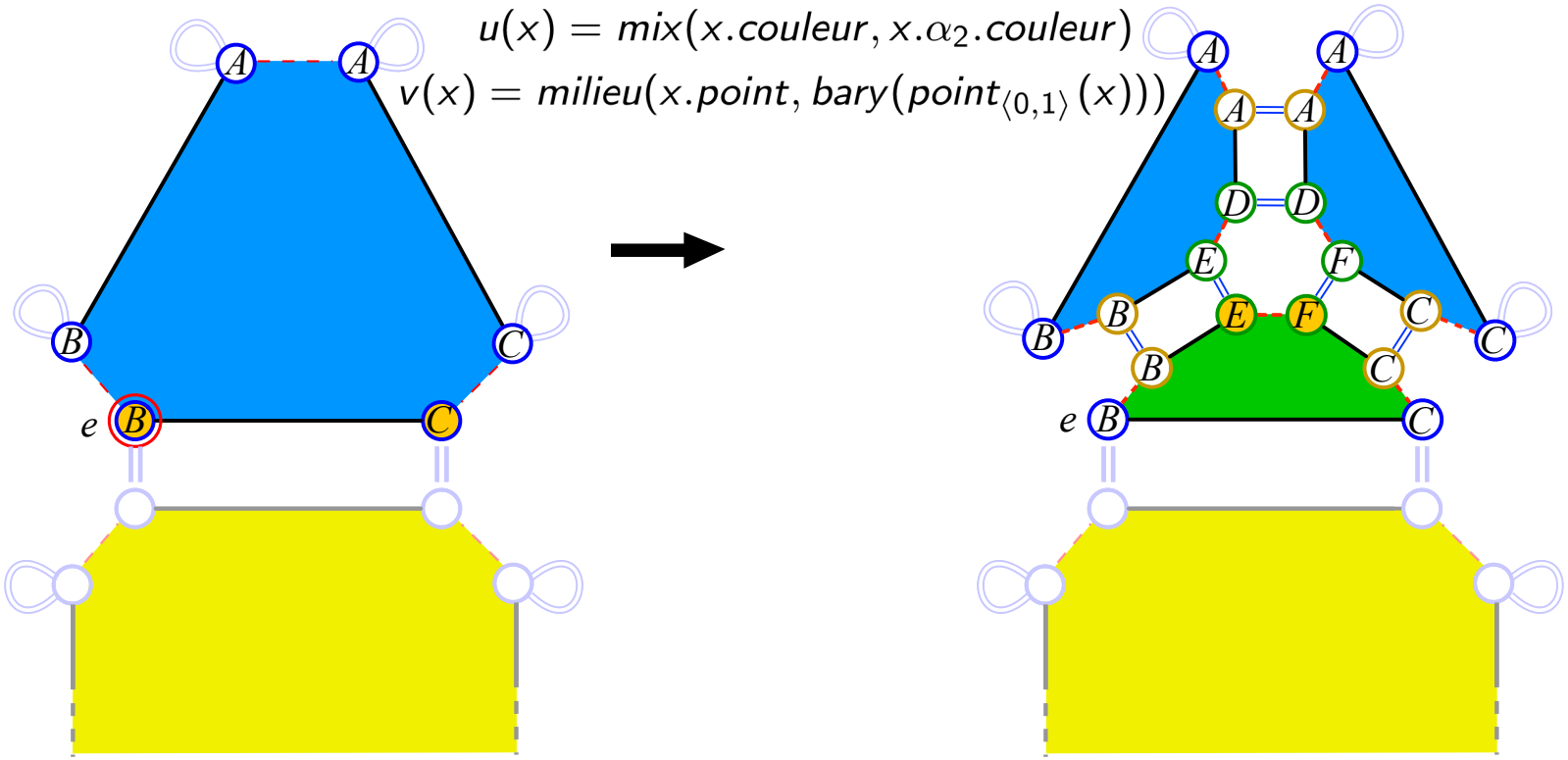
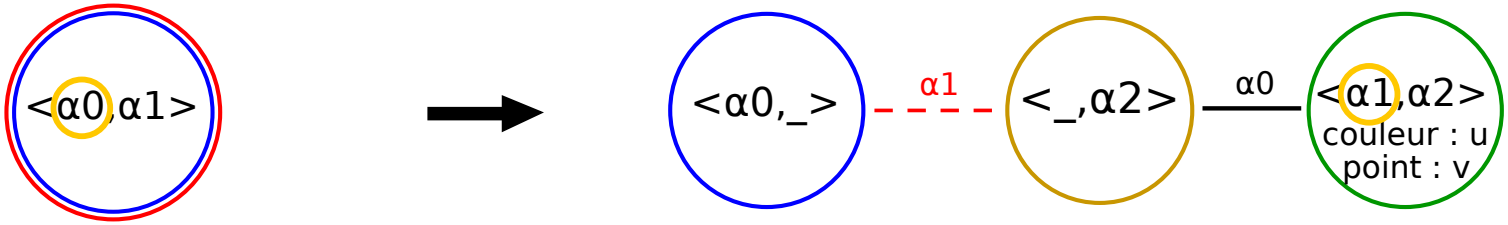


$x \equiv_{\langle 1,2\rangle} x.\alpha_0$ et $bary(point_{\langle 0,1\rangle}(x)) \equiv_{\langle 1,2\rangle} bary(point_{\langle 0,1\rangle}(x.\alpha_0))$

Condition



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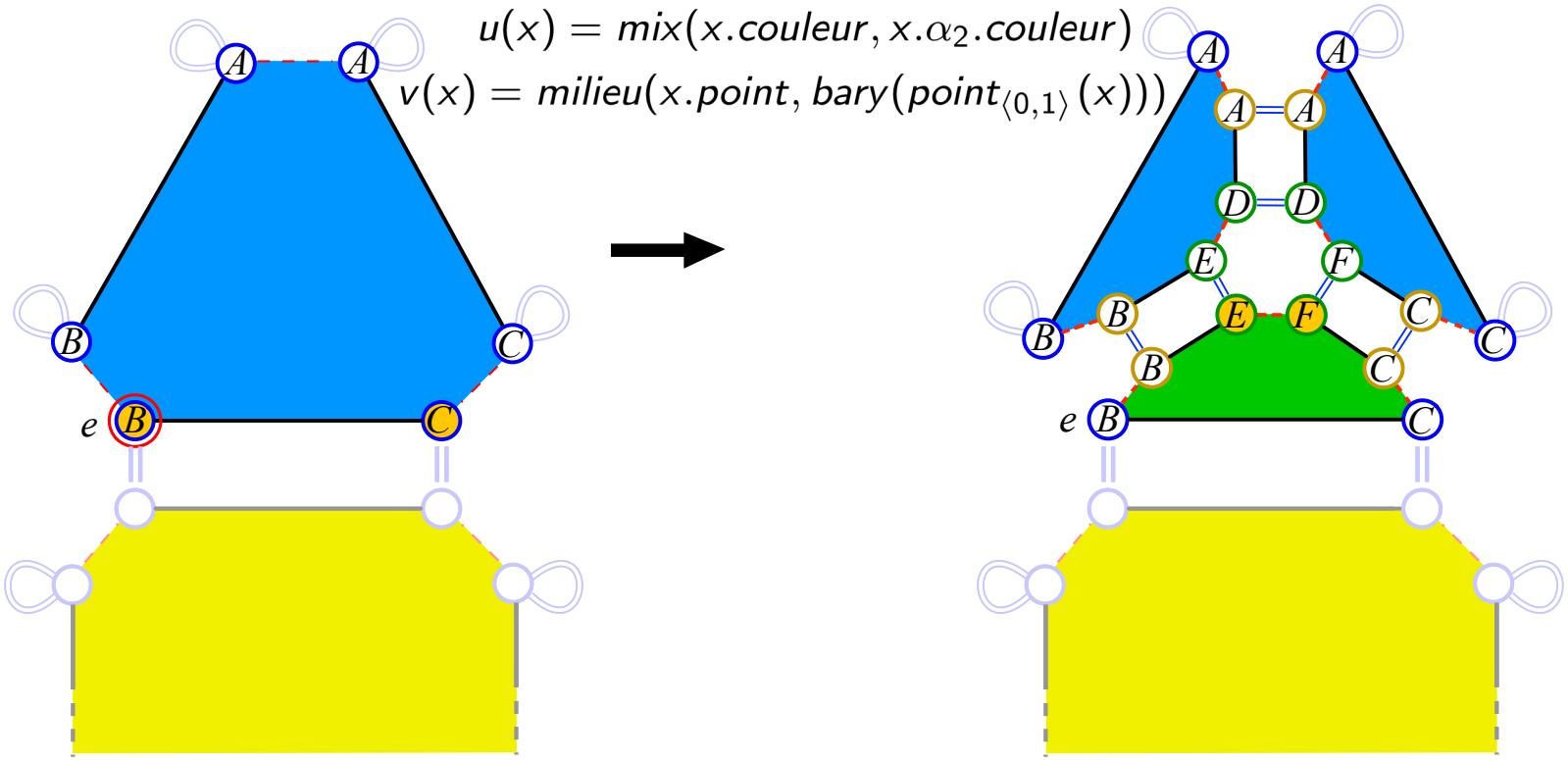


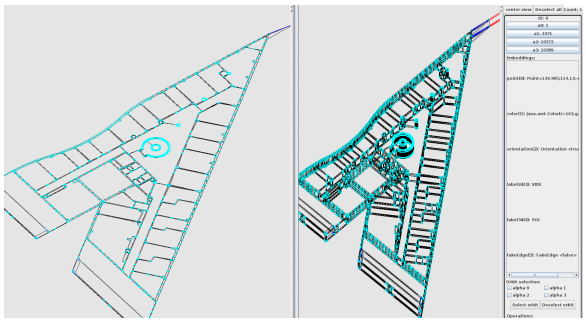
$x \equiv_{\langle 1,2\rangle} x.\alpha_0$ et $bary(point_{\langle 0,1\rangle}(x)) \equiv_{\langle 1,2\rangle} bary(point_{\langle 0,1\rangle}(x.\alpha_0))$

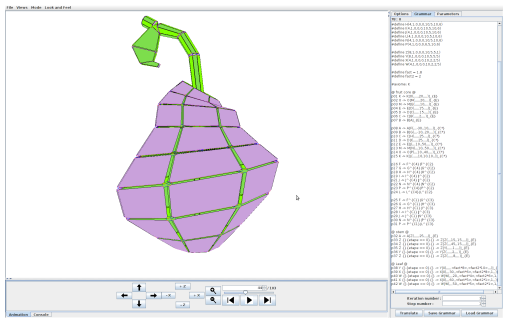
Bilan

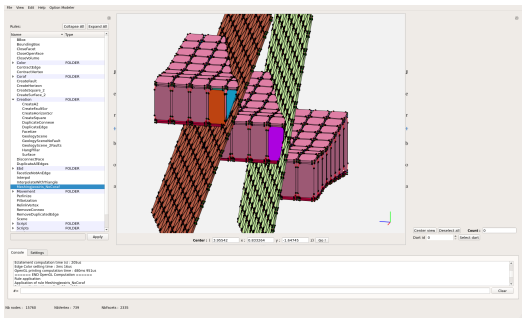
- Définition d'un langage d'expression
- Vérifications syntaxiques : préservation de la cohérence de plongement

Perspectives

- Intégrer les vérifications dans l'éditeur
- Définir un langage de composition des règles







Questions

Merci de votre attention